

# Optimal food price stabilisation policy\*

Christophe Gouel<sup>†</sup>

October 28, 2011

## Abstract

This paper proposes a framework for designing optimal food price stabilisation policies in a self-sufficient developing country. It uses a rational expectations storage model with risk-averse consumers and incomplete markets. Government stabilises food prices by carrying public stock and by applying state-contingent subsidy/tax to production. The policy rules are designed to maximise intertemporal welfare. The optimal policy under commitment crowds out all private stockholding activity by removing the profit opportunity from speculation. The countercyclical subsidy/tax to production helps price stabilisation by subsidising production in periods of scarcity and by taxing it in periods of glut. It contributes little to welfare gains, most of which come from stabilisation achieved through public storage.

*Keywords:* food price stabilisation, incomplete markets, risk-aversion, storage.

*JEL classification:* D52, Q11, Q18.

---

\*I would like to thank Jean-Marc Bourgeon, Benjamin Carton, Édouard Challe, Christopher Gilbert, Alexandre Gohin, Sébastien Jean, Michel Juillard, and Brian Wright for helpful comments. Support from the AgFoodTrade project funded under the 7<sup>th</sup> Framework Programme for Research and Development, DG-Research, European Commission, is acknowledged. The views expressed here are the sole responsibility of the author and do not reflect those of the Commission or of the World Bank.

<sup>†</sup>The World Bank and CEPII (cgouel@worldbank.org)

# 1 Introduction

The turmoil in agricultural markets in the 2007–08 food crisis and its 2010–11 sequel has served as a stark reminder of the inherent instability of agricultural markets. In reaction, many countries struggle to secure their supply through foreign land acquisition (von Braun and Meinzen-Dick, 2009) or try to increase their self-sufficiency through input subsidy programmes (Dorward, 2009).

This context may be new but intervening for stabilising food markets is age-old and widespread in developing and emerging countries. For example, China and India operate very large stabilisation programmes (Dorosh, 2008). In China, government controls most international grain trade and carries large stocks (in the 2000s Chinese stocks of cereals—corn, rice, wheat—represented 40% of world stocks). India operates the world’s largest public food distribution system, covering about 600 million people: government builds stocks from procurements at minimum support prices and sells subsidised rations. The Egyptian government subsidises cooking oil, sugar, bread and flour (Löfgren and El-Said, 2001). Bread is also subsidised in Iran, involving an outlay of 6.5% of government expenses in 2001 (Amid, 2007). Given the pervasiveness of these policies and the potential cost of food price spikes for poor consumers, it is essential to identify the precise economic motives for intervention and to design the policies accordingly.

This is the object of this paper, which defines a framework for studying food price stabilisation policies in a self-sufficient developing country. We build a model representing the main features of food price volatility. The model includes a market imperfection justifying public intervention: market incompleteness associated with consumer risk aversion. The model features the behaviours of a competitive storer, a producer exposed to productivity shocks, and a risk-averse consumer. Prices are stabilised through an optimal policy under commitment using as instruments a countercyclical subsidy/tax on planned production and a public stock.

Our analysis provides three main findings. First, provided that consumers are risk averse, there is an optimal food price stabilisation policy that improves social welfare. The optimal policy consists of a public storage rule and a subsidy/tax on planned production, which works like a set-aside programme. The management of public stock is such that it removes any profit opportunity from speculation and crowds out all private storage. Second, producers enjoy short-run gains because the launch of the policy is accompanied by a phase of transitional stock build-up which pushes up prices. In the long-run, producers may lose because stabilisation entails lower prices on average. Third, the countercyclical subsidisation of production contributes little to welfare gains, most of which come from stabilisation through public storage.

This paper has relevance beyond the study of food price stabilisation policies. Its combination of methods for designing optimal dynamic policies, widely used in macroeconomics for fiscal or monetary policy analysis, with a rational expectations storage model, able to mimic the behaviour of agricultural markets, could permit, with the appropriate developments, the design of food policies such as contingent cash transfers to consumers or input subsidy programmes in a context of stochastic prices and yields.

Commodity price stabilisation schemes have been studied before (for a survey, see Wright, 2001), but mostly with the purpose of analysing agricultural policies, such as deficiency payments or floor-price programmes (Miranda and Helmberger, 1988, Wright and Williams, 1988a, Glauber et al., 1989, Gardner and López, 1996). These works impose exogenous policy rules in the rational expectations storage model, which, although very stylised, is able to reproduce the most important features of commodity markets (Cafiero et al., 2011).<sup>1</sup> A setting common to most studies based on this model is perfect market equilibrium, which implies that there is no rationale for public intervention, and stabilising markets beyond what is done by private agents is a waste of public money.

One exception is the work of Gardner (1979) who considers that periods of high food prices generate external costs that justify public intervention. To increase welfare, he designs a discretionary optimal stockpiling policy. He also discusses price-band policies and analyses the uncertainties that could hinder the implementation of such policies. Unlike Gardner (1979), the present paper is more explicit about the source of market failure in considering market incompleteness, which is in line with the abundant literature on the cost of commodity price instability against which consumers are assumed to be unable to insure (Waugh, 1944, Turnovsky et al., 1980, Helms, 1985b, Wright and Williams, 1988b). It also makes a methodological contribution since, in addition to solving the model numerically, it follows the modern literature on optimal dynamic policies (Marcet and Marimon, 2011) in allowing the derivation of first-order conditions for an optimal dynamic policy, which delivers interpretable conditions.

Another approach to this issue is that adopted by Newbery and Stiglitz. Their approach departs from the framework of the infinite-horizon storage model. Newbery (1989) shows that market incompleteness and consumer risk aversion may justify government intervention in the form of public storage or distribution of food rations. This study enlarges on his previous work with Stiglitz, which considers the role of public intervention in incomplete commodity markets (Newbery and Stiglitz, 1981, 1982, 1984). Their work exploits two-period models, which cannot account for the stylised facts of food prices. For example, commodity prices are positively serially correlated (Deaton and Laroque, 1992), so storage management must account for the fact that high prices may be followed by high prices, making it suboptimal to dump all the stock in one period, as would be recommended by Newbery's model. More generally, two-period models allow a single optimal stockpile size to be identified, whereas any applications based on a realistic framework require an optimal storage rule that stipulates how stock should be accumulated or sold given the prevailing market conditions. Such a storage strategy is the result of an infinite-horizon problem, as in the present paper. Overall this paper combines Newbery and Stiglitz's market incompleteness assumption with a storage model calibrated on values typical of the cereals market in a developing country, and uses tools from optimal policy design to propose stabilisation policies.

The organisation of the paper is as follows. Section 2 describes the storage model without public policy. Section 3 presents the optimal policy approach. We define a social welfare function that serves as policy objective, and then derive and analyse the first-order conditions of the optimal policy

---

<sup>1</sup>Its traditional form is presented and analysed in depth by Wright and Williams (1982a).

problem. In Section 4, we calibrate the model and analyse the numerical results, discussing the various decision rules, then separately analysing transitional and asymptotic behaviours. Section 5 concludes.

## 2 The model

Analysis of food price stabilisation policy should take account of the origin of food price volatility and reproduce its main features. The model used in most of the work on commodity price stabilisation schemes, such as Miranda and Helmberger (1988) and Wright and Williams (1988a), is the single-country competitive storage model. Cafiero et al. (2011) recently proved that this model performs well for explaining international commodity price behaviour.

Our framework extends this model to account for consumer risk aversion, market incompleteness, and government stabilisation policy. Time is discrete. This partial equilibrium model features an annual market for a storable commodity with a competitive storer, a producer whose output is subject to multiplicative shocks, and a final demand.

Despite the fact that international prices are an important driver of the domestic price in many countries, our focus is on a closed economy and we assume that domestic productivity shocks are the only source of dynamics. Agricultural markets are the object of many protectionist policies (Anderson, 2010), including self-sufficiency (e.g., cereals in China and India), making the autarkic case a not uncommon situation.<sup>2</sup>

### 2.1 Consumers

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification:  $D(P_t, Y) = dP_t^\alpha Y^\eta$ , where  $d > 0$  is a normalisation parameter;  $P_t$  is the period  $t$  price;  $Y$  is income, which is assumed to be constant; and  $\alpha$ , with  $\alpha < 0$  and  $\alpha \neq -1$ , and  $\eta \neq 1$  are the price and income elasticities. Assuming there are only two goods and the second good is the numeraire, the integration of this demand function gives the following instantaneous indirect utility function (Hausman, 1981)

$$\hat{v}(P_t, Y) = \frac{Y^{1-\eta}}{1-\eta} - d \frac{P_t^{1+\alpha}}{1+\alpha}. \quad (1)$$

This utility function has a relative risk aversion equal to the income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a): we assume  $\hat{v}(P_t, Y)$  to be positive and apply a monotone transformation to the indirect utility function,

$$v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1+\theta}, \quad (2)$$

---

<sup>2</sup>In a companion paper, Gouel and Jean (2011) extend this framework to an open economy setting and analyse optimal stabilisation policies in an small open country.

with  $v(P_t, Y) \rightarrow \ln \hat{v}(P_t, Y)$  as  $\theta \rightarrow -1$ . This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is then

$$\rho(P_t, Y) = \eta - \theta \frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}, \quad (3)$$

with  $\theta$  indexing the degree of risk aversion.<sup>3</sup>

For the sake of simplicity, the representative consumer is assumed to adopt a hand-to-mouth behaviour: he consumes current income and does not save to smooth out fluctuations. The dynamics are thus simplified, since consumer's "cash on hand" does not have to be included as a state variable. This assumption overlooks the role of self-insurance through saving. However, such self-insurance remains limited in practice and falls short of providing protection comparable to what complete markets deliver, due *inter alia* to borrowing constraints and to the rather large budget share of staple food in many developing countries.

Given the absence of saving, the consumer does not solve an intertemporal problem. At each period, the consumer is concerned only with current-period demand, which is not affected by the degree of risk aversion. So speculators face the same profit opportunities if the economy is populated by risk-averse or risk-neutral consumers. This absence of effect of risk aversion on demand creates the need for public intervention, since private storers do not account for it.

## 2.2 Storers

There is a single representative speculative storer, which is risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity  $S_t^s$  from period  $t$  to period  $t + 1$  entails a purchasing cost,  $P_t S_t^s$ , and a storage cost,  $k S_t^s$ , with  $k$  the unit physical cost of storage. In addition, a share  $1 - \delta$  of the commodity deteriorates during storage. The benefits in period  $t$  are the proceeds from the sale of previous stocks:  $\delta P_t S_{t-1}^s$ . The storer follows a storage rule that maximises its expected profit as stated by

$$V^S(S_{t-1}^s, P_t) = \max_{\{S_{t+i}^s \geq 0\}_{i=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i [\delta P_{t+i} S_{t+i-1}^s - (P_{t+i} + k) S_{t+i}^s] \right\}, \quad (4)$$

where  $E_t$  denotes the mathematical expectations operator conditional on information available at time  $t$  and  $\beta$  is the discount factor. The storer's problem can be expressed in a recursive form using the following Bellman equation:

$$V^S(S_{t-1}^s, P_t) = \max_{S_t^s \geq 0} \{ \delta P_t S_{t-1}^s - (P_t + k) S_t^s + \beta E_t [V^S(S_t^s, P_{t+1})] \}. \quad (5)$$

---

<sup>3</sup>Following the same method, other demand curves could have been contemplated. Few, however, have an indirect utility function with a known form. In such case, the numerical algorithm of Vartia (1983) can be used to find a numerical approximation of the indirect utility function.

This equation has two state variables: the price, whose dynamics is considered by the storer to be exogenous, and the stock carried over from the previous year. Using the first-order condition on  $S_t^s$  and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition<sup>4</sup>

$$S_t^s \geq 0 \quad \perp \quad \beta \delta E_t (P_{t+1}) - P_t - k \leq 0, \quad (6)$$

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. So this is a situation of a stabilising speculation, the storer buys when prices are low and when he rationally expects that they will be higher later.

### 2.3 Producers

A representative producer makes his productive choice one period before bringing output to market. He puts in production in period  $t$  a level  $H_t$  for period  $t + 1$ , but a multiplicative disturbance affects final production (e.g., a weather disturbance). The producer chooses the production level by solving the following maximisation of expected profit:

$$\max_{\{H_{t+i}\}_{i=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i [P_{t+i} \epsilon_{t+i} H_{t+i-1} + \zeta_{t+i} H_{t+i} - \Psi(H_{t+i})] \right\}, \quad (7)$$

where  $\Psi(H_t)$  is the cost of planning the production  $H_t$  and  $H_t \epsilon_{t+1}$  is the realised production level.  $\epsilon_{t+1}$  is the realisation of an i.i.d. stochastic process of mean 1 exogenous to the producer, which follows a translated beta distribution.  $\zeta_t$  is a state-contingent subsidy/tax on planned production, received by producers at planting time, which is considered later as a possible stabilisation tool. Like the storer's problem, this problem can be reformulated into a recursive form, and its solution gives the following Euler equation:

$$\beta E_t (P_{t+1} \epsilon_{t+1}) + \zeta_t = \Psi'(H_t). \quad (8)$$

This equation has a straightforward interpretation: it is the equality between the marginal cost of production and the expected discounted marginal benefit of one unit of planned production. The production cost function is assumed to be convex, as increasing production requires increasing the use of less fertile lands, and to follow an isoelastic form

$$\Psi(H) = h \frac{H^{1+\mu}}{1+\mu}, \quad (9)$$

---

<sup>4</sup>Complementarity conditions in what follows are written using the ‘‘perp’’ notation ( $\perp$ ). This means that the expressions on either side of the sign are orthogonal. If one equation holds with strict inequality, the other must have an equality.

where  $h > 0$  is a scale parameter and  $\mu \geq 0$  is the inverse of supply elasticity.

## 2.4 Recursive equilibrium

At the beginning of each period, three predetermined variables define the state of the model:  $S_{t-1}^S$ ,  $H_{t-1}$  and  $\epsilon_t$ . When government pursues food price stabilisation through public storage, public carry-over, noted  $S^G$ , must also be included to define the state of the model. Public stocks are assumed to deteriorate at the same rate as private stocks. These four variables can be combined in one state variable, availability, the sum of production, and private and public carry-over:

$$A_t = H_{t-1}\epsilon_t + \delta (S_{t-1}^S + S_{t-1}^G). \quad (10)$$

Market equilibrium can be written as<sup>5</sup>

$$A_t = D(P_t) + S_t^S + S_t^G. \quad (11)$$

From the above, we can define the recursive equilibrium of the problem without public policies:

**Definition.** In the absence of stabilisation policy (i.e.,  $\zeta = S^G = 0$ ), a recursive equilibrium is a set of functions,  $S^S(A)$ ,  $H(A)$  and  $P(A)$ , defining storage, production and price over the state  $A$  and the transition equation (10) such that (i) storer solves (4), (ii) producer solves (7), and (iii) the market clears.

## 3 Optimal policy approach

We assume that government is able to commit at the first period to following a policy rule that maximises expected intertemporal welfare (the so-called Ramsey policy). We design the optimal policy by following the modern literature on optimal policy in dynamic economies (Marcet and Marimon, 2011). As the policy is conceived to correct for the inability of consumers to insure against food price volatility, the first-best policy would be to mimic the role of the futures market by providing state-contingent cash payments to consumers. In the real world, few policies work in this way, but some can be related to this logic. Food and cash-for-work schemes, for example, are labour-intensive work programmes offering in kind or cash payments (Barrett, 2002). They are designed to combat food insecurity and to target those suffering adverse shocks. Other frequent policies in the developing world are food price stabilisation policies. Because of their practical importance, we focus on these second-best policies, which target price behaviour. Here, the stabilisation instruments are subsidy/tax on planned production and public stocks, where government is assumed to be able to manage storage facilities at the same costs as private storers.

---

<sup>5</sup>As income is assumed to be constant, in what follows demand function is only expressed as a function of price.

The policy starts at period 0 and is unanticipated by the agents. The parameters of the stabilisation policy are determined by maximising a social welfare function that aggregates consumer utility, other agents' surpluses and fiscal cost. Initial availability is taken as being at its deterministic steady-state level and initial private stocks are assumed to be null.

### 3.1 Social welfare function

Working with a partial equilibrium model imposes some constraints. For example, it is not possible to take the expected sum of discounted consumer's utility as the objective of the policy authority as is often done in optimal policy problems. Instead, all effects on the welfare of other agents must be included in the objective. In a general equilibrium model, there is no need for this since the consumers' income includes all earnings. The approach adopted here also differs from most partial equilibrium analyses due to its consideration of risk aversion. In partial equilibrium models, policy design is based on maximising the sum of all agents' surpluses. This does not take account of risk aversion, though, since the expected consumer surplus is not a measure of the welfare of risk-averse agents (Helms, 1985b, Stennek, 1999).

Instead of using the sum of agents' surpluses, we have to define the preferences of the government. We assume that government tries to maximise a social welfare function that weights linearly the welfare of each agent. We want to leave aside all distributional considerations. For this purpose, the determination of weights has to ensure that transfers between agents do not affect welfare. The weights should remove the transfers and leave in the social welfare function only efficiency gains from stabilisation and the cost of the policy. At period  $t$ , social welfare is given by:

$$W_t = v(P_t, Y) + w_H [P_t H_{t-1} \epsilon_t + \zeta_t H_t - \Psi(H_t)] + w_S [\delta P_t S_{t-1}^S - (P_t + k) S_t^S] - w_C Cost_t, \quad (12)$$

where  $w_H$ ,  $w_S$  and  $w_C$  are the weights given in social welfare to producers' income, storers' realised profit and fiscal cost; and  $Cost_t$  is the fiscal cost of the policy. Since producers and storers are both assumed to be risk neutral, if government has no distributional bias, there is no reason to weight their profits differently, so their weights can be considered equal,  $w_S = w_H$ . Defined in this way, the social welfare function shows that the optimal policy will consist in a risk-sharing arrangement between risk-averse consumers and risk-neutral agents, government in particular.

To rule out transfers between agents, we need to know how the consumer values a monetary transfer in utility terms. For a given price  $P$ , a small monetary transfer  $\tau$  to the consumer increases utility by approximately (at the first order)  $v_Y(P, Y) \tau$ . Thus, we could use the marginal utility with respect to income as the weight. But, given that the price is stochastic, we have to consider the marginal utility of income over all the price distribution and not just for one price. The computation is based on the ergodic distribution of price without public policies:

$$w_H = E[v_Y(P, Y)]. \quad (13)$$

This is the unconditional expectation of marginal utility over income.<sup>6</sup>

Although also a monetary value, fiscal cost does not necessarily have the same weight as agents' profit. Unless raised by a lump-sum tax, income raised for public expenses entails distortionary costs due to revenue collection. We ignore these costs and assume  $w_C = w_H$ . This assumption can be removed, but turns out to be convenient as described below.

The cost of the policy is the cost of carrying public stock plus the cost of subsidising planned production. The cost of public storage is similar to the profit from private storage. It is equal to the difference between the purchasing plus the storage costs of new stock and the revenue derived from selling previous stock,  $\delta S_{t-1}^G$ :

$$Cost_t = (P_t + k) S_t^G - \delta P_t S_{t-1}^G + \zeta_t H_t. \quad (14)$$

Using  $w$  as the unique weight of monetary terms in social welfare, we can write social welfare as

$$W_t = v(P_t, Y) + w [P_t A_t - \Psi(H_t) - (P_t + k)(S_t^S + S_t^G)]. \quad (15)$$

Here the common money measure is used to sum the predetermined terms,  $H_{t-1}\epsilon_t$ ,  $S_{t-1}^S$  and  $S_{t-1}^G$ , to obtain current availability,  $A_t$ . This means that it does not matter for the welfare evaluation who holds the availability; whether it is in the producers' hands, in private stock or in public stock is a matter of indifference. As a result, production, private and public stock need not be considered as separate state variables. They are combined in one state variable, which is very convenient for the numerical solution since complexity increases exponentially with the number of state variables.<sup>7</sup>

### 3.2 Optimal policy problem

The government maximises the expected sum of the discounted instantaneous social welfare function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{v(P_t, Y) + w [P_t A_t - \Psi(H_t) - (P_t + k)(S_t^S + S_t^G)]\}, \quad (16)$$

subject to the constraints imposed by private agents' behaviour and market equilibrium. In the previous equation, it is assumed that the consumer's rate of time preference is equal to the discount parameter,  $\beta$ , which corresponds to a discount by the real interest rate. This equality does not

---

<sup>6</sup>This weighting scheme takes as reference the situation without intervention, so it is defined in an equivalent variation approach. In a compensating variation logic, the weight would be different and would be equal to the unconditional expectation of marginal values of utility over income on the ergodic distribution of price after stabilisation. Numerically, they do not differ much, but the compensating variation approach poses the problem that the welfare weight is endogenous to the optimal policy determination.

<sup>7</sup>Other social welfare functions could have been contemplated (McLaren, 1997, Slesnick, 1998, Im, 2001). Agricultural and food policies are often designed to serve particular interests such as those of the urban poor in developing countries or of farmers in developed countries, alternative weighting schemes could reflect such distributional biases. The weighting scheme chosen here, however, has the advantage to focus on efficiency effects; it obeys the compensation principle and leads to easily interpretable equations.

generally occur, but this assumption avoids the notational clutter of having to use two parameters to discount utility and monetary terms separately.

Among the constraints imposed by private agents' behaviour, the first-order condition for the storers' behaviour (6) is a complementarity equation and cannot enter directly as constraint in a maximisation problem. To restate this complementarity equation, we introduce a positive slack variable,  $\phi$ , with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta\delta \mathbf{E}_t (P_{t+1}), \quad (17)$$

$$S_t^s \phi_t = 0. \quad (18)$$

$\phi$  refers to the marginal loss from storage. If positive, arbitraging by storage entails a loss in expectation, and so there is no private storage. If it is null, prices are arbitraged between the two successive periods.

By using equations (8), (10), (11), (17) and (18) as constraints to the maximisation problem, introducing the corresponding present-time Lagrange multipliers, and applying the law of iterated expectations, the dynamic optimisation problem with its Lagrangian can be defined as follows:

$$\begin{aligned} \min_{\{\Phi_t\}_{t=0}^{\infty}} \max_{\{\Omega_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & v(P_t, Y) + w[P_t A_t - \Psi(H_t) - (P_t + k)(S_t^s + S_t^g)] \\ & + \lambda_t (\beta\delta P_{t+1} + \phi_t - P_t - k) \\ & + \kappa_t S_t^s \phi_t \\ & + \nu_t [\beta P_{t+1} \epsilon_{t+1} + \zeta_t - \Psi'(H_t)] \\ & + \chi_t [A_t - D(P_t) - S_t^s - S_t^g] \\ & + \xi_t [A_{t+1} - \delta(S_t^s + S_t^g) - H_t \epsilon_{t+1}] \}, \end{aligned} \quad (19)$$

where  $\Phi_t = \{\lambda_t, \kappa_t, \nu_t, \chi_t, \xi_t\}$ ,  $\Omega_t = \{S_t^s \geq 0, H_t, P_t, S_t^g \geq 0, \phi_t \geq 0, \zeta_t, A_{t+1}\}$ , and  $A_0$  is given.

From the first-order conditions, and after some manipulation we get the following system of complementarity conditions, which define the dynamics of the optimal policy under commitment

$$S_t^s : S_t^s \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta\delta \mathbf{E}_t (wP_{t+1} + \chi_{t+1}) + \kappa_t \phi_t \leq 0, \quad (20)$$

$$H_t : \beta \mathbf{E}_t (\chi_{t+1} \epsilon_{t+1}) - w\zeta_t - \nu_t \Psi''(H_t) = 0, \quad (21)$$

$$P_t : v_P(P_t, Y) + wD(P_t) - \lambda_t + X_t - \chi_t D'(P_t) = 0, \quad (22)$$

$$S_t^g : S_t^g \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta\delta \mathbf{E}_t (wP_{t+1} + \chi_{t+1}) \leq 0, \quad (23)$$

$$\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \kappa_t S_t^s \leq 0, \quad (24)$$

$$\lambda_t : \phi_t = P_t + k - \beta\delta \mathbf{E}_t (P_{t+1}), \quad (25)$$

$$\kappa_t : S_t^s \phi_t = 0, \quad (26)$$

$$\zeta_t : \nu_t = 0, \quad (27)$$

$$\nu_t : \beta \mathbf{E}_t (P_{t+1} \epsilon_{t+1}) + \zeta_t = \Psi' (H_t), \quad (28)$$

$$\chi_t : A_t = D (P_t) + S_t^S + S_t^G, \quad (29)$$

with  $\lambda_{-1} = \nu_{-1} = 0$  since government surprises other agents in starting the stabilisation policy, hence it is not constraint by past private expectations. Transition equations are (10) and

$$X_t = \delta \lambda_{t-1} + \nu_{t-1} \epsilon_t. \quad (30)$$

Corresponding to the two forward-looking equations (17) and (8), two Lagrange multipliers,  $\lambda$  and  $\nu$ , appear as lagged variables in the first-order conditions. As expressed in (30), they are summed together in the state variable  $X$ . The introduction of these lagged variables as new state variables gives the problem its recursive form (Kydland and Prescott, 1980, Marcet and Marimon, 2011). Government, in its decisions, has to account for private agents' forecasts of its own future actions. Indeed in a rational expectations equilibrium, government confirms the earlier expectations of private agents. The need to satisfy private expectations makes the problem history-dependent. Government choice depends on both the natural state variable,  $A$ , and the history of the state. The role of the lagged multipliers is to summarise this trajectory by measuring the social cost of confirming past expectations in current behaviour.

### 3.3 Discussion of first-order conditions

The first-order conditions can be interpreted as follows. The first-order condition on public storage (23) is an arbitrage equation in the utility metric. While similar to the arbitrage equation of private storage without policy (6), it differs in one important aspect: public storage arbitrages the total marginal values of the commodity across two consecutive periods, while private storage arbitrages only on the private marginal values. The total marginal value of the commodity is the sum of its social marginal value, summarised by the Lagrange multiplier on market clearing equation,  $\chi$ , and its private marginal value (i.e., price) converted in the utility metric,  $wP$ .

Equation (27) states that the Lagrange multiplier on the producers Euler equation is zero. It means two things, which are related. First, the condition on producers' behaviour does not constrain welfare maximisation and, second, the forward-looking behaviour of producers does not imply any time-inconsistencies for the optimal policy. More specifically, a technical distinction between a policy with a credible precommitment and a discretionary policy would be that in the former case the policy maker can manipulate expectations, but not in the latter. Here, the policy maker can use an instrument equivalent to manipulating the producers' expectations since he can choose the level of the subsidy, which, in terms of the producers' behaviour, has the same incentive effect as changing expectations.

Using  $\nu_t = 0$ , the first-order condition on production (21) can be written as

$$\zeta_t = \beta \mathbf{E}_t (\chi_{t+1} \epsilon_{t+1}) / w, \quad (31)$$

which has a straightforward interpretation. The subsidy is equal to the discounted expected marginal social value in the money metric of one unit of planned production. This implies that production is no longer planned with respect to prices only, but is based on total marginal values.

Private storers' behaviour is hard to identify in the first-order conditions. So we rely on intuitions confirmed in the numerical analysis carried in the next section. Private and public stocks have perfectly symmetric roles in the social welfare function, market equilibrium and definition of availability, since, in these equations, they are always summed. As policy instruments they have the same effects on price dynamics, even though private storage imposes an additional constraint: private storers are profit-seeking and behave according to equations (17)–(18). Since the two instruments are equivalent, but private storage imposes constraints, the maximisation should privilege public storage, and as long as public storage leads to higher stock level than private storage it should crowd out this latter.<sup>8</sup>

In the absence of private storage, price behaviour can be characterised through equation (23), by reformulating it as

$$P_t = \max [D^{-1}(A_t), \beta\delta E_t(P_{t+1} + \chi_{t+1}/w) - \chi_t/w - k]. \quad (32)$$

This equation defines a two-regime behaviour for the price function: for low availability there is no storage, price is defined by the inverse of the demand function at current availability (first term). For higher availability, public storage links current price to the future marginal value of the commodity.

The issue of commitment comes from the expectations of private agents. Producers' expectations are indirectly under the control of the subsidy/tax on planned production. And since, under an optimal policy, speculative storers do not stock anything, their expectations are irrelevant for public decision. Hence, the problem can be simplified. Without private stock, equation (17) does not impose any constraint and  $\lambda = X = 0$ . The state variable  $X$  is introduced to account for the effect of lagged multipliers on current decision. These multipliers being null, the problem has one state variable: availability. The crowding out of private storage implies that there is no commitment problem. The public decision would be identical under commitment and under discretion.

This absence of commitment problem is specific to the combination of the instruments implemented. The government takes control of all intertemporal decisions, so it does not face any commitment problem. If only one instead of two instruments of stabilisation is considered, the distinction between commitment and discretion re-emerges. Stabilisation through public storage without production-related policy would crowd out private storage, but producers' expectations of future public actions would create a time-inconsistency problem. The same applies to a policy using only the subsidy/tax on planned production: storers' expectations affect the time-consistency of the policy. Since in analysing numerical results it is important to identify the contribution of each policy instrument

---

<sup>8</sup>There is one situation where there is no crowding out: the situation of a risk-neutral consumer. In this case, there is no rationale for public intervention and the optimal policy is to stockpile exactly as would be done by the private storer alone. Here, equations (17)–(18) do not impose constraints since the optimal stock level is the profit maximising one (Scheinkman and Schechtman, 1983).

to stabilisation, in the following section, we simulate the model with each instrument separately. From the previous system of equations, an optimal public storage policy is obtained by removing equation (27) and all appearances of  $\zeta$ . An optimal policy of a subsidy/tax to production is the solution of the previous system without equation (23) and terms in  $S^G$ .

Equation (22) gives some insights into the behaviour of the social marginal value of the commodity. Using  $\lambda = X = 0$  and Roy's identity, this equation gives

$$\chi_t = \frac{P_t}{\alpha} [w - v_Y(P_t, Y)]. \quad (33)$$

The behaviour of the social marginal value can be discussed by comparing  $w$  with  $v_Y$ .  $w$  is the average value of marginal utility over income for the price distribution without stabilisation (13). If the stabilisation is not too important (and the cost of stabilisation policy will prevent it), the marginal utility over income should still fluctuate around  $w$  and the sign of  $\chi_t$  should alternate. For a relative risk aversion superior to income elasticity,  $v_Y$  increases with price.<sup>9</sup> So the social marginal value of the commodity is positive for high prices, and correspondingly low availability, and negative for low prices, and correspondingly high availability. This implies that the optimal policy should try to move the commodity from periods of high availability, when the social marginal value is negative, to periods of low availability, when it is positive. This discussion sheds some light on the behaviour of the subsidy/tax on production as defined by (31). When the next-period availability is expected to be high, which implies that a large stock is carried over, producers are taxed in order to limit supply in a period with negative social marginal value. On the contrary, for low stock levels, the next-period availability is expected to be low and production is subsidised.

## 4 Numerical results

The model without stabilisation policy, and a fortiori the model with optimal policy, cannot be solved analytically. Decision rules must be approximated numerically. The problems are solved under Matlab using a projection method with a collocation approach. The solver is similar to Fackler (2005), except that the equilibrium equations are solved by the mixed complementarity problems solver PATH (Dirkse and Ferris, 1995).<sup>10</sup> A grid on the state variables is chosen (using 50 points for availability and 15 for lagged Lagrange multipliers), on which decision rules are approximated by splines and the number of grid points is chosen such that the use of these decision rules entails on average less than a \$1 error every \$1,000 of decision (measured by the Euler equation error).

---

<sup>9</sup>By differentiating Roy's identity with respect to income and by using the definition of income elasticity and relative risk aversion, we have  $v_{YP} = v_{YY}D(\eta/\rho - 1)$ .

<sup>10</sup>For more precisions, see the RECS solver (<https://github.com/christophe-gouel/rececs>), with which the model is solved.

## 4.1 Calibration

Parameterisation is required in order to quantify the welfare effects of introducing a food stabilisation policy. Table 1 presents the parameters used to calibrate the model. The parameters are set such that, at the model’s deterministic steady state, price, production, consumption and availability are equal to 1. Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share,  $1/\gamma$ . An annual interest rate of 5% is used for discounting.

**Table 1. Parameterisation**

Parameter	Economic interpretation	Assigned value
$\beta$	Annual discount factor	0.95
$\eta$	Income elasticity	0.5
$\alpha$	Own-price demand elasticity	-0.4
$\gamma$	Commodity budget share	0.15
$\mu$	Inverse of supply elasticity	10
$Y$	Income	6.67
$d$	Normalisation parameter of demand function	0.39
$h$	Normalisation parameter of production cost function	0.95
$\theta$	Parameter defining risk aversion	-2.62
$k$	Physical storage cost	0.06
$\delta$	Share of commodity remaining after one year of storage	0.98
$\epsilon$	Probability distribution of yield	$B(2, 2) \cdot 0.5 + 0.75$

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: -0.4 for price elasticity and 0.5 for income elasticity. For poor households, the budget share devoted to one commodity can be important. For example, rice expenditures represent roughly 20–35% of total expenditure for the bottom quintile in Bangladesh, India, Indonesia and Philippines (for a share of food expenditures around 62–70%, Asian Development Bank, 2008). For the top quintile, it is about 5–10% of total expenditure. Since the main focus of a food security policy is the poorest populations, the representative consumer is assumed to spend, at the steady state, 15% of its income on the staple.

We assume at the steady state a relative risk aversion parameter of 2, implying  $\theta = -2.62$ . For this calibration, the third derivative of indirect utility with respect to price,  $v_{PPP}$ , is negative. Given that utility decreases with price,  $v_{PPP} < 0$  implies an aversion to downside price risk (Eeckhoudt and Schlesinger, 2006). For a given mean and standard deviation, the consumer prefers a price distribution with higher third moment, that is, shifting the volatility toward low rather than high prices. It matters for the design of stabilisation policies, because storage tends to skew prices upwards.

Brennan (2003) indicates that the annual physical cost of storage for rice in Bangladesh is around 6% of the value of rice. Assuming a per-unit storage cost of 6% of the steady-state price gives

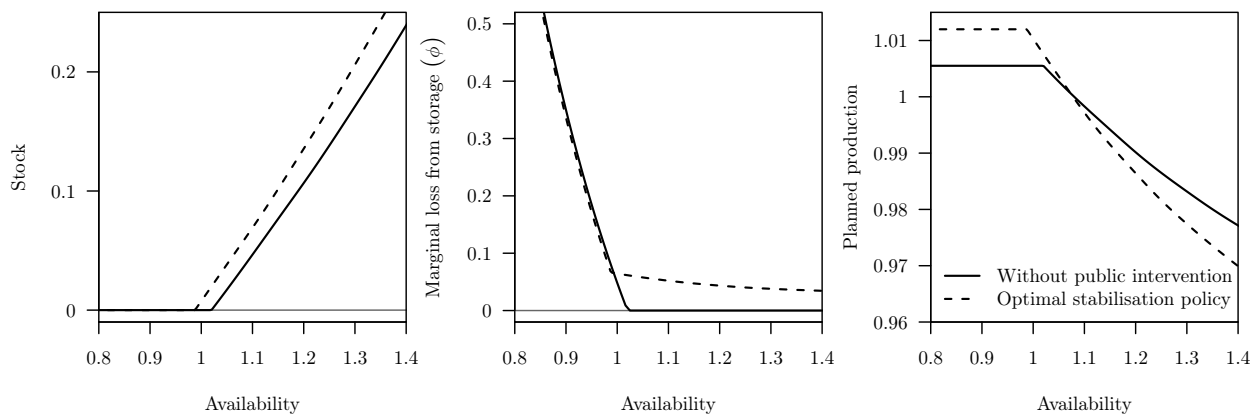
$k = 0.06$ . Assuming 2% of storage depreciation ( $\delta = 0.98$ ), the overall storage cost—opportunity cost, physical storage cost and depreciation—at steady-state is 14% of the steady-state price.

Supply elasticity is set at 0.1, so  $\mu = 10$ . This value is confirmed by Roberts and Schlenker (2010) who analyse a supply and demand model of world commodities, and estimate global supply elasticity for the caloric equivalent of corn, rice, soybean and wheat between 0.08 and 0.13. FAPRI’s elasticities database<sup>11</sup> confirms this order of magnitude. For example, for developing countries they propose supply elasticity for wheat between 0.07 and 0.43. The productive shocks,  $\epsilon$ , are assumed to follow a beta distribution of shape parameters, 2 and 2, which makes it unimodal at 0.5 and symmetric. The distribution is recentred and rescaled to vary between 0.75 and 1.25, which implies a coefficient of variation of 11.2%.

## 4.2 Decision rules

### 4.2.1 Storage rules

Storage rules are presented in Figure 1 (left panel). Without public policy, private storers (solid line) do not stock anything for low availability (the threshold is close to the steady-state availability level, 1), and increase the level of stock with market availability above the threshold. When normal consumption is satisfied, any additional quantity in the market tends to lower prices. The speculator takes opportunity of these lower prices to accumulate stock that can be sold in periods of lower availability. The middle panel shows the behaviour of the marginal loss from storage,  $\phi$ . It is positive for low availability, which explains the absence of storage: profit is negative. As soon as stock is positive, marginal loss is null. Private storers operate in expectations at zero profit.



**Figure 1. Storage, marginal loss from storage and production rules**

Stabilisation through an optimal public storage rule (dashed line) results in levels of public stock that are always superior to the levels reached by private storage without policy. In consequence,

<sup>11</sup><http://www.fapri.iastate.edu/tools/elasticity.aspx> (accessed May, 2010).

public stocks discourage any speculative storage, and the marginal loss from storage is always positive. The crowding-out of private storage means that the degree of involvement of government in storage has to go beyond the difference between the private storage rule without intervention and the optimal rule. Because public storage removes profit opportunity, public intervention means government is responsible for both speculative storage and additional storage motivated by the lack of market for sharing the risk borne by consumer. This complete crowding-out is absent in the optimal storage rules in Wright and Williams (1982b) and Williams and Wright (1991, Ch. 15), which analyse the management of strategic petroleum reserves. Two features explain the coexistence of both public and private stocks in these works: in the first study, private storers are assumed to receive a convenience yield from the holding of stocks, implying that they hold stock even if the apparent return is negative; in the second study, they suppose that public stock is not held at the same location as private stock, which gives justification for the existence of private storers that are closer to the market. For these reasons, and because private storers hold stocks to smooth the natural seasonality of agriculture production, it is reasonable to think an optimal public storage policy would not in practice completely crowd out private storage. But there will be very little scope for private storage to obey a speculative motive in the presence of welfare-maximising public storage.

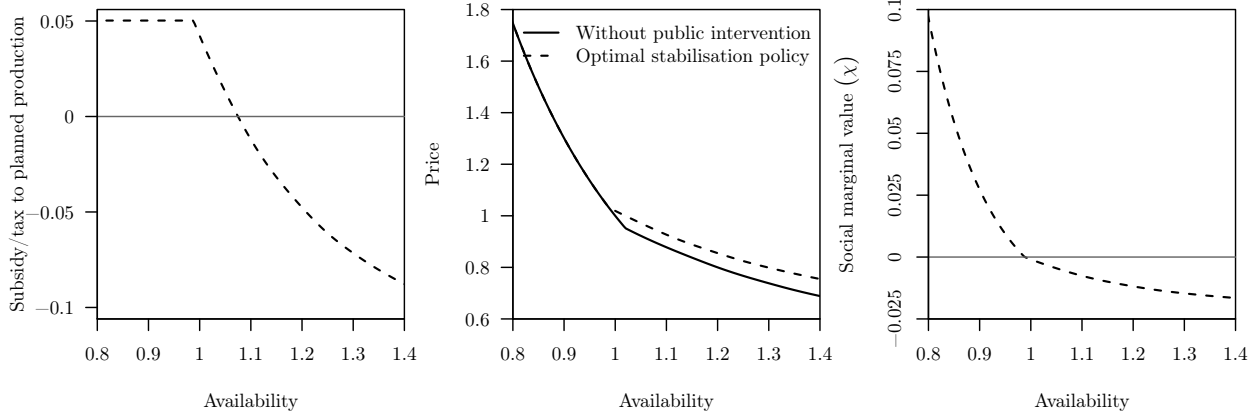
Since there is a complete crowding out, the stabilisation brought by public storage could also be decentralised through a state-contingent subsidy to private storage. The subsidy would consist in covering the marginal loss from storage induced by the higher stock level to bring the expected private storers' profit to zero; hence the subsidy would follow  $\phi$  as represented in Figure 1.

#### 4.2.2 Production rules

Like the storage rule, the production rule presents a kink at the stockout limit (Figure 1, right panel). When storage is null, whatever the current availability, the planned production level is the same. For positive stock, the next-period expected price decreases with availability and so too does planned production.

The effect of public policy is to make production more elastic to current availability. Production is higher for low availability and lower for high availability. This is achieved through a subsidy to planned production for low availability and a tax for high availability (see Figure 2, left panel). For low availability levels, the storage policy is ineffective, since there will be no stockpiling. But it is possible to increase future availability by producing more. This is the point of subsidising planned production.

A state-contingent subsidy policy makes sense only for a storable commodity. Without storage, the rational expectations solution to the model without public intervention is a constant planned production level (Muth, 1961). Storage creates variation in the expected price and therefore justifies the variation in production with growers producing more when availability is expected to be low.



**Figure 2. Subsidy/tax to planned production, price rules, and social marginal value of the commodity**

The subsidy/tax magnifies this effect beyond private behaviour. In essence, this instrument plays a complementary role to storage.

It may seem strange to tax producers in abundant years when they are suffering low prices, but taxation is the incentive part of the policy; it is possible to compensate producers for any loss through non-distortive transfer. It is possible also to design policy such that producers are not taxed but are paid to take land out of production. These types of policies were common in the US as part of supply management until the 1996 FAIR Act (Gardner, 2002, Ch. 7). The US Secretary of Agriculture decided on the percentage of land that farmers should set aside each year, in order to be eligible for the price support programme. This policy was seen as complementing storage policies, since it limited production when stocks were already high.

The supply control through subsidies and taxes to planned production may, however, be more difficult to implement in a developing country where a direct control of producers' behaviour is much more difficult, given the administrative system required to verify acreages. An alternative instrument, which could partly mimic the role of a supply control policy, is a state-contingent fertiliser subsidy, since fertiliser subsidies have been common in many African countries (Morris et al., 2007).

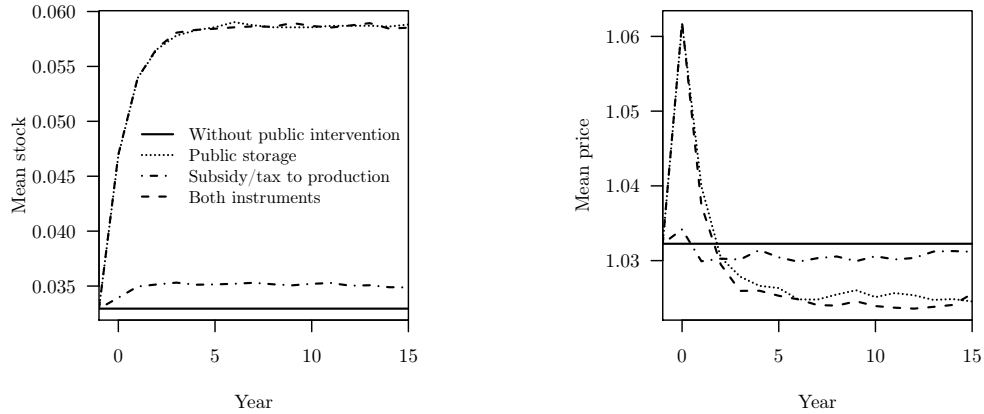
### 4.2.3 Price and social marginal value of the commodity

The two regimes, without and with storage, are also apparent in the price rules (Figure 2, middle panel). In a stockout situation, price is determined solely by current consumer demand and can increase a lot for small availability. For higher availability, demand for storage adds to consumer demand, which makes total demand more elastic and limits the decrease in the price with availability. Under an optimal policy, for positive stocks, the price is higher because demand for storage is higher. The behaviour of the social marginal value,  $\chi$ , is represented in Figure 2 (right panel). It is positive for low availability, meaning it is socially justifiable to try to transfer more resources to

low availability periods, hence the higher level of stock. The social marginal value is negative for high availability, which explains taxation of production in these situations in order to prevent excess supply.

### 4.3 Dynamics under optimal stabilisation policy

When stabilisation policy is first introduced, the new asymptotic distribution is not reached immediately, so we analyse, in this subsection, the transitional behaviour following policy implementation. In addition to the effects of an optimal policy with two instruments, the results from an optimal public storage policy on its own, and an optimal subsidy/tax to planned production are depicted. Since a storage policy involves a higher mean stock level than without intervention, it begins with a phase of stock build-in. The transition is illustrated in Figure 3 with the dynamics of mean total stock and mean prices when the policy starts with the system on the asymptotic distribution without public intervention.



**Figure 3. Transitional dynamics.** Before period 0, the system is on the asymptotic distribution of the model without public intervention. The unexpected policy is announced and starts in period 0. (Obtained by Monte Carlo simulation as the average over 100,000 simulated paths.)

In the first years, the purchases made to accumulate stocks push the mean price above its long-run mean. After a few years, the mean price drops below its value without public intervention. Stabilisation affects the mean price through the curvature of the inverse demand function (Wright, 1979). Since we apply constant elasticity of demand, the inverse demand function is convex. Storage also makes the inverse of the total demand function convex, as shown in Figure 2. Thus, the price at mean consumption is lower than the mean price. So stabilisation of the consumption results in a lower mean price.

The dynamic welfare effects of the policy are presented in Table 2. Consumer gains are *ex ante* per-period equivalent variation,  $EV$ , calculated using the following implicit definition

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [v(P_t, Y + EV) - v(\check{P}_t, Y)] \right\} = 0, \quad (34)$$

where  $P$  and  $\tilde{P}$  are the prices before and after stabilisation. Producers' gains are the annualised changes in expected profit from equation (7). Government outlays are the annualised expected sum of the discounted costs from equation (14). Changes in storers' surpluses are ignored since storers operate at zero profit on average and we have assumed no stock in the first period.<sup>12</sup> To put the magnitude of the welfare results in perspective, they can be compared to income or to the steady-state commodity budget share. Total gains from the optimal policy using both instruments represent 0.014% of income and 0.094% of the commodity budget share. The low level of these gains is not surprising given that the gains resulting from a reduction of risk are of second order. The finding parallels Lucas's (1987) insight that individuals would be unlikely to sacrifice more than 0.1% of their lifetime consumption to live in a world with no macroeconomic volatility. The low level of these welfare gains is discussed below in the sensitivity analysis and in conclusion.

**Table 2. Welfare results on transitional dynamics** (as a percentage of the steady-state commodity budget share.)

	Public storage	Subsidy/tax to production	Both instruments
Consumers gains	0.33	0.30	0.48
Producers gains	0.03	1.09	0.66
Government outlays	0.27	1.37	1.05
Public storage	0.27	–	0.26
Subsidy/tax to production	–	1.37	0.80
Total gains	0.09	0.02	0.09

Stabilisation policies result in transfers between agents (see Newbery and Stiglitz, 1981, Ch. 6 and 9, for a discussion of efficiency and transfer from price stabilisation). Consumers gains include two components: a transfer term corresponding to the change in mean expenditures, and an efficiency term corresponding to the risk reduction and the change in mean consumption. The mean expenditure change is a transfer with producers, and private and public storers. Public storage entails both storage costs and changes in mean price, i.e., a transfer with producers and consumers. The subsidy/tax on production is a transfer between government and producers. Overall, the transfers sum to zero and do not affect social welfare, which is composed only of efficiency gains from consumers, and changes in production and storage costs.

For a policy of public storage, stockpiling pushes prices up in the early periods and in later periods pushes them below their mean value without stabilisation. For producers, this implies long-term losses from lower prices. However, since later periods are discounted, overall producers gain from a stockpiling policy. These producers' gains stem mainly from transfers from government through the effect of public storage on mean price.

Producers' gains are higher under a pure subsidy/tax policy. This policy is a combination of subsidy in periods of low availability and tax in periods of abundance. But, without public storage, subsidies

<sup>12</sup>These welfare results stem from expected infinite sums of discounted values (e.g., equation (7)). To calculate them, we follow the dynamic programming trick of transforming the infinite sums in recursive problems that can be solved easily by value function iteration.

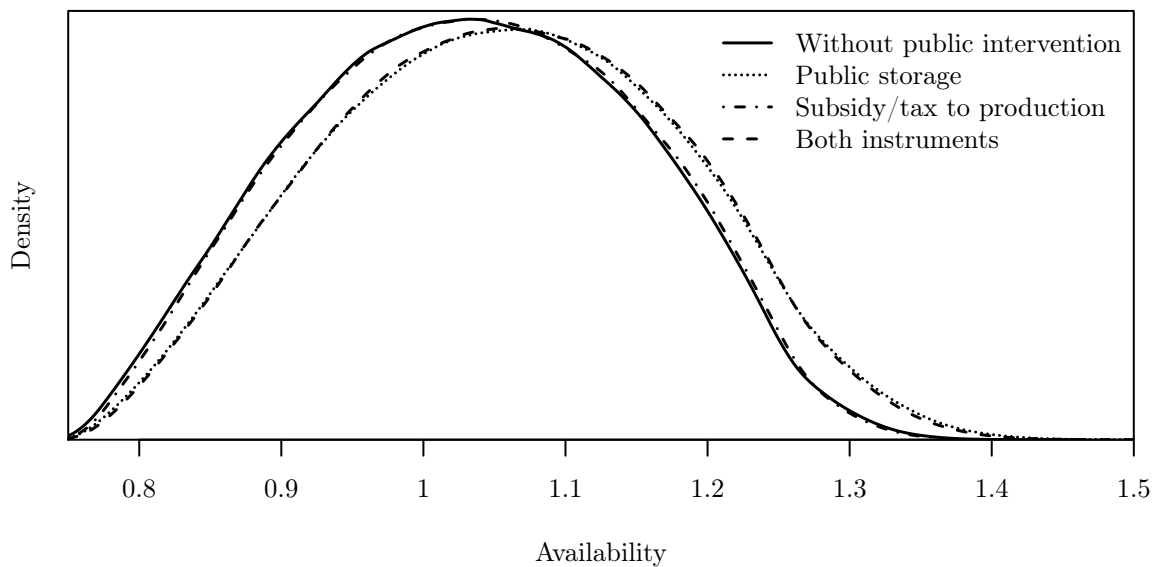
are more frequent than taxes, and producers benefit greatly from this instrument. Under this policy, situations of low availability are more frequent than when both instruments apply. It is precisely when availability is low that the producers are subsidised to produce more, which explains why producers gain more when output subsidy is the only policy.

The sign of the change in producers surplus is sensitive to the intensity of the stabilisation policy, which is governed by the levels of risk aversion and commodity budget share. We report below the sensitivity analysis with respect to these parameters and discuss the reason for this dependence.

Having accounted for all transfers, public storage contributes the most to overall welfare. A policy of planned production subsidy only complements public storage.

#### 4.4 The long-run effects of stabilisation

Since it is not possible to borrow the commodity for the future (the non-negativity constraint on storage), increased stabilisation as a result of public storage requires an increase in mean stock levels, as stock dynamics show (Figure 3). Following this transitory phase of stock building availability increases due to the higher stock levels as is apparent from the asymptotic distribution of availability (Figure 4).<sup>13</sup> Because public storage shifts the distribution of availability toward higher values, it reduces the risk of shortfalls. Subsidising planned production acts differently on availability. It reduces the probability of low and high availability by making supply more elastic, although the effect appears marginal.



**Figure 4. Density of availability.** The curves are obtained by kernel density estimation over 1,000,000 simulated points.

<sup>13</sup>Long-run results are calculated over 1,000,000 sample observations from the asymptotic distribution.

Confirming previous results, Table 3 shows that all stabilisation policies involving public storage decrease the mean price. This decrease brings long-run losses for producers, because stabilisation decreases mean profit (Turnovsky, 1976, shows that this is always true for a constant-elasticity demand where elasticity is below unity).

**Table 3. Descriptive statistics of price series on the asymptotic distribution**

	Benchmark	Public storage	Subsidy/tax to production	Both instruments
Mean	1.033	1.025	1.031	1.024
Coefficient of variation	0.233	0.199	0.226	0.196
Skewness	1.288	1.609	1.595	1.314
Quantiles				
1%	0.749	0.777	0.762	0.788
25%	0.860	0.889	0.864	0.890
50%	0.940	0.965	0.940	0.964
75%	1.155	1.087	1.148	1.086
99%	1.777	1.722	1.757	1.708

Most of the stabilisation is achieved through public storage, which on its own decreases the coefficient of variation from 0.233 to 0.199. Introducing in addition state-contingent subsidy decreases it to 0.196. Storage stabilises by moving the commodity from periods of low prices to periods of high prices. This is apparent in the fact that the probability of the occurrence of low and high prices decreases. The possibility of stockouts means that high prices are not all alleviated, and the skewness of prices increases with the level of storage.

The quantiles of prices show how storage policies operate. Because the public storage rule is more aggressive than the private storage rule, low price periods are less frequent since government uses these opportunities to build up stocks. More than half of the distribution ends up at higher prices, but the higher stock level allows the occurrence of high prices to reduce.

#### 4.5 Nature of the risk-sharing arrangement

The stabilisation policies increase total welfare by transferring risk from risk-averse consumers to risk-neutral agents. Table 4 illustrates this risk sharing by presenting the standard deviation of each component of the social welfare function (12) on the asymptotic distribution: consumers' utility, producers' and storers' profit and government outlays.

**Table 4. Standard deviation of social welfare components**

	Benchmark	Public storage	Subsidy/tax to production	Both instruments
Consumers utility	0.033	0.028	0.032	0.027
Producers profit	0.123	0.106	0.159	0.125
Storers profit	0.049	0	0.052	0
Government outlays	0	0.065	0.059	0.053

As price volatility decreases with public intervention, the standard deviation of consumers' utility decreases for all policies. The effects on producers' profit are more contrasted. A public storage policy decreases the volatility of their profit, but the other policies increase it. The subsidy/tax has a procyclical behaviour with respect to producers' profit. In periods of low availability, producers' profit is high because the high prices more than compensate for the poor harvests; this is also the period when they receive subsidies to increase production. And conversely, they are taxed to limit production when their profit is the lowest, when availability is high. This increase in profit volatility creates no welfare loss in our model since producers are assumed to be risk neutral. This assumption is useful in limiting the number of state variables,<sup>14</sup> but it is not realistic. Given that a policy of subsidy/tax alone increases significantly the volatility of producers' profit, this policy would not be followed as such. This is less true when this instrument is associated with public storage, since the volatility of profit increases only slightly. It does not imply, however, that a supply control policy is useless with risk-averse producers.<sup>15</sup> Its precise form as stemming from the optimal policy design would just be different.

With regard to risk sharing, public storage appears the most balanced policy. It decreases the volatility perceived by both consumers and producers, and transfers the risk to government budget. The public storage policy found here would not be optimal with risk-averse producers, but it would probably increase social welfare, and the welfare of both consumers and producers.

#### 4.6 Sensitivity to risk aversion and budget share

The risk aversion and the budget share devoted to the commodity are two parameters that have important implications for the quantitative findings. These two parameters govern the importance of the welfare losses from price instability. Consumers enjoy welfare gains from a stabilisation at mean price if their indirect utility function is concave in price. From Turnovsky et al. (1980), we know that this is the case if  $\gamma(\eta - \rho) - \alpha < 0$ , which illustrates the role of these parameters.

As the budget share is increased, the stabilisation policy becomes more aggressive and achieves a lower level of price instability (see Table 5). There are also more welfare gains to expect from an optimal policy with a higher budget share. This and the above formula from Turnovsky et al. (1980) indicate that a price stabilisation policy may have quite differentiated impacts with heterogeneous households. Rich households, which devote a tiny fraction of their budget to staple food, are likely to suffer from a stabilisation at mean price, as they benefit from the instability by consuming more at low prices and by limiting the nuisance of high prices by consuming less (this is the traditional results of Waugh, 1944). In Table 5, consumers gain from stabilisation when their budget share is equal to 1% of their income not because of their risk aversion, but because of the slight mean price

---

<sup>14</sup>With risk-averse producers, it would not have been possible to sum production and stocks into one state variable, availability. The increase in dimension would have made the problem more difficult to solve and the results more difficult to interpret.

<sup>15</sup>The frequent association of export control with buffer stock in international commodity agreements (Gilbert, 1996) is an indication that controlling supply was thought as complementing the stockholding operations.

decrease. On the contrary, for poor households, the benefit from stabilisation may be substantial. Part of it comes from the mean price decrease, but the contribution of the risk premium reduction is also important.

**Table 5. Sensitivity to risk aversion and budget share**

Risk aversion ( $\rho$ )	0.5	1			2			4		
Budget share ( $\gamma$ )	0.15	0.01	0.15	0.3	0.01	0.15 <sup>a</sup>	0.3	0.01	0.15	0.3
<i>Welfare results</i>										
Consumers gains	-0.00	0.01	0.20	0.36	0.04	0.48	0.74	0.10	0.84	1.19
Producers gains	-0.00	0.03	0.35	0.54	0.08	0.66	0.52	0.18	0.52	-1.44
Government outlays	0.00	0.04	0.53	0.85	0.12	1.05	0.92	0.27	0.93	-1.66
Total gains	-0.00	0.00	0.01	0.04	0.00	0.09	0.34	0.00	0.44	1.41
<i>Asymptotic statistics</i>										
Mean price	1.033	1.033	1.029	1.026	1.032	1.024	1.022	1.031	1.022	1.029
CV of price	0.233	0.232	0.219	0.206	0.230	0.196	0.168	0.226	0.162	0.124

*Notes:* Results for an optimal policy with both instruments. Welfare results expressed as percentages of the steady-state commodity budget share.

<sup>a</sup> Benchmark.

The results do not evolve linearly with both parameters. For instance, for high budget share and high risk aversion, producers tend to lose from the policy, whereas they gain for other parameters' values. Producers' surplus is affected differently by the two policy instruments. Public storage leads to an initial phase of stock build-in, which pushes price up in the first periods. Prices then go down because of the convexity of the demand function. The discounting of future losses makes that producers gain from public storage. The production policy is a combination of subsidy in times of scarcity and taxes in times of glut. If the stabilisation policy is aggressive and leads to high mean stock level, situations of glut are more frequent than situations of scarcity and producers are taxed in average. If this taxation exceeds the gains from the initial price increase, they lose from the policy. This is what we observe in the last column of Table 5. The negative government outlays is a confirmation of this behaviour, since it indicates that government is actually making money with this policy, and this can only come from the taxation of producers. In addition, the non-monotone behaviour of mean price with respect to budget share is an outcome of the two countervailing forces: mean price decrease caused by the stabilisation of consumption with a convex demand and mean price increase caused by the lower production level stemming from the taxation.

## 5 Conclusion and perspectives

In this paper we proposed a framework for food price stabilisation policy design. This framework takes some inspiration from the method used in the modern literature on optimal monetary policy (Clarida et al., 1999): (i) we start with a simple model, the rational expectations storage model,

that is able to account for the stylised facts of commodity prices; (ii) because markets are incomplete, consumers are unable to insure themselves, which could justify public intervention; (iii) corresponding to the market failure, a public policy maximising the welfare is designed.

This work brings together two approaches. That proposed by Newbery and Stiglitz, which uses very stylised models to propose stabilisation policies in imperfect markets, but does not result in implementable policies due to the restrictive framework, and the tradition of competitive storage modelling (e.g., Miranda and Helmberger, 1988, Wright and Williams, 1988a) in which public policies are discussed within a framework that mimics the main features of commodity markets, but neglects market imperfections. The resulting model, although still highly stylised, provides a theoretical benchmark against which food security propositions can be assessed.

We show that an optimal policy under commitment composed of state-contingent public storage and state-contingent subsidy/tax on planned production improves social welfare and crowds out all private storage activity by removing any profit opportunity from speculation. Most of the welfare gains are from public storage. A countercyclical production policy, which provides incentives for increased production in times of scarcity and decreased production in times of abundance, makes little difference and can have undesirable consequences such as increasing the volatility of producers' profit.

By choice, we kept the model as simple as possible. Future work could extend this study along a number of important dimensions. One important issue is the size of welfare gains. They are low, below 1% of the commodity budget share, and consequently food price stabilisation policies may not be considered as highly desirable. There are, however, good reasons, outside the scope of this paper, to expect higher gains. For instance, the welfare cost stemming from risk aversion accounts only for a static effect and food price instability generates adjustments that have dynamic effects creating irreversibilities in household welfare, such as removing children from school (de Janvry et al., 2006) or decreasing food consumption to the extent that it has long-run health consequences (Chen and Zhou, 2007). In addition, our modelling of food demand is very simple and does not acknowledge the fact that food consumption cannot decrease too much without consequences for health or even death. Accounting for the specific role of food would reinforce the need to prevent the occurrence of high food prices. This could be done with the help of the survival function proposed by Ravallion (1987) or the microeconomic framework developed in Chavas (2000). Assigning to food a peculiar role as a consumption good should increase the welfare gains from stabilisation beyond the low values found here. These effects may justify more aggressive stabilisation policies, but are delicate to integrate in a welfare framework and, thus, have been neglected in this paper and in previous works on the welfare cost of food price instability.

Also, the optimal policy with state-contingent public storage and subsidy to planned production implies heavy government involvement in the functioning of markets and the application of highly nonlinear policy rules. Historical experience of the involvement of states in grain markets shows mixed results and several instances of large inefficiencies (Dorosh, 2008). It would be useful to

consider the design of simpler policies that would decentralise a part of the policy implementation to the private sector.

Finally, we do not allow for heterogeneity among consumers. If people likely to suffer from high prices represent only a limited share of the total population, it may not be optimal to engage in large-scale public stock programmes. A better scheme might include policies specifically targeting people in need. They might take the form of targeted cash transfer programmes (Skoufias et al., 2001) or the distribution of subsidised rations. The public stock backing a targeted policy would not necessarily crowd out private storage since it would not be designed to affect the whole market.

## References

- Amid, J. (2007). The dilemma of cheap food and self-sufficiency: The case of wheat in Iran. *Food Policy*, 32(4), 537–552.
- Anderson, K. (2010). Krueger, Schiff, and Valdes revisited: Agricultural price and trade policy reform in developing countries since 1960. *Applied Economic Perspectives and Policy*, 32(2), 195–231.
- Asian Development Bank (2008). *Food Prices and Inflation in Developing Asia: Is Poverty Reduction Coming to an End?* Special report, ADB, Economics and Research Department, Manila.
- Barrett, C. B. (2002). Food security and food assistance programs. In B. L. Gardner and G. C. Rausser (eds.) *Agricultural and Food Policy*, volume 2B, part 5 of *Handbook of Agricultural Economics*, chapter 40, (pp. 2103–2190). Amsterdam: Elsevier.
- von Braun, J. and Meinzen-Dick, R. (2009). “*Land Grabbing*” by Foreign Investors in Developing Countries: *Risks and Opportunities*. Policy Brief 13, IFPRI.
- Brennan, D. (2003). Price dynamics in the Bangladesh rice market: implications for public intervention. *Agricultural Economics*, 29(1), 15–25.
- Caffero, C., Bobenrieth, E. S. A., Bobenrieth, J. R. A. and Wright, B. D. (2011). The empirical relevance of the competitive storage model. *Journal of Econometrics*, 162(1), 44–54.
- Chavas, J.-P. (2000). The microeconomics of food security. *The Australian Journal of Agricultural and Resource Economics*, 44(1), 1–29.
- Chen, Y. and Zhou, L.-A. (2007). The long-term health and economic consequences of the 1959–1961 famine in china. *Journal of Health Economics*, 26(4), 659–681.
- Clarida, R., Galí, J. and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature*, 37(4), 1661–1707.
- Deaton, A. and Laroque, G. (1992). On the behaviour of commodity prices. *Review of Economic Studies*, 59(1), 1–23.
- Dirkse, S. P. and Ferris, M. C. (1995). The PATH solver: A non-monotone stabilization scheme for mixed complementarity problems. *Optimization Methods and Software*, 5, 123–156.

- Dorosh, P. A. (2008). Food price stabilisation and food security: International experience. *Bulletin of Indonesian Economic Studies*, 44(1), 93–114.
- Dorward, A. (2009). Rethinking agricultural input subsidy programmes in a changing world, paper prepared for the Trade and Markets Division, Food and Agriculture Organization of the United Nations.
- Eeckhoudt, L. and Schlesinger, H. (2006). Putting risk in its proper place. *The American Economic Review*, 96(1), 280–289.
- Fackler, P. L. (2005). A MATLAB solver for nonlinear rational expectations models. *Computational Economics*, 26(2), 173–181.
- Gardner, B. L. (1979). Robust stabilization policies for international commodity agreements. *The American Economic Review*, 69(2), 169–172, Papers and Proceedings of the Ninety-First Annual Meeting of the American Economic Association.
- Gardner, B. L. (2002). *American Agriculture in the Twentieth Century. How It Flourished and What It Cost*. Cambridge: Harvard University Press.
- Gardner, B. L. and López, R. (1996). The inefficiency of interest-rate subsidies in commodity price stabilization. *American Journal of Agricultural Economics*, 78(3), 508–516.
- Gilbert, C. L. (1996). International commodity agreements: An obituary notice. *World Development*, 24(1), 1–19.
- Glauber, J. W., Helmerger, P. G. and Miranda, M. J. (1989). Four approaches to commodity market stabilization: A comparative analysis. *American Journal of Agricultural Economics*, 71(2), 326–337.
- Gouel, C. and Jean, S. (2011). Optimal food price stabilisation in a small open developing country, paper presented at the 23rd Annual Bank Conference on Development Economics, May 30–June 1, Paris.
- Hausman, J. A. (1981). Exact consumer’s surplus and deadweight loss. *The American Economic Review*, 71(4), 662–676.
- Helms, L. J. (1985a). Errors in the numerical assessment of the benefits of price stabilization. *American Journal of Agricultural Economics*, 67(1), 93–100.
- Helms, L. J. (1985b). Expected consumer’s surplus and the welfare effects of price stabilization. *International Economic Review*, 26(3), 603–617.
- Im, J.-B. (2001). An optimal storage rule with political preferences toward interest groups. *The Korean Economic Review*, 17(2), 377–392.
- de Janvry, A., Finan, F., Sadoulet, E. and Vakis, R. (2006). Can conditional cash transfer programs serve as safety nets in keeping children at school and from working when exposed to shocks? *Journal of Development Economics*, 79(2), 349–373.
- Kydland, F. E. and Prescott, E. C. (1980). Dynamic optimal taxation, rational expectations and optimal control. *Journal of Economic Dynamics and Control*, 2, 79–91.

- Löfgren, H. and El-Said, M. (2001). Food subsidies in Egypt: reform options, distribution and welfare. *Food Policy*, 26(1), 65–83.
- Lucas, R. E., Jr. (1987). *Models of Business Cycles*. Oxford, England: Basil Blackwell, Yrjo Jahnsson Lectures.
- Marcet, A. and Marimon, R. (2011). *Recursive Contracts*. Discussion Paper 1055, CEP.
- McLaren, J. (1997). *Speculatively Constrained Optimal Commodity Policies*. Discussion Paper 9697-27, Columbia University.
- Miranda, M. J. and Helmlinger, P. G. (1988). The effects of commodity price stabilization programs. *The American Economic Review*, 78(1), 46–58.
- Morris, M., Kelly, V. A., Kopicki, R. J. and Byerlee, D. (2007). *Fertilizer Use in African Agriculture. Lessons Learned and Good Practice Guidelines*. Washington: The World Bank.
- Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica*, 29(3), 315–335.
- Newbery, D. M. G. (1989). The theory of food price stabilisation. *The Economic Journal*, 99(398), 1065–1082.
- Newbery, D. M. G. and Stiglitz, J. E. (1981). *The Theory of Commodity Price Stabilization: A Study in the Economics of Risk*. Oxford: Clarendon Press.
- Newbery, D. M. G. and Stiglitz, J. E. (1982). Risk aversion, supply response, and the optimality of random prices: A diagrammatic analysis. *The Quarterly Journal of Economics*, 97(1), 1–26.
- Newbery, D. M. G. and Stiglitz, J. E. (1984). Pareto inferior trade. *Review of Economic Studies*, 51(1), 1–12.
- Ravallion, M. (1987). Towards a theory of famine relief policy. *Journal of Public Economics*, 33(1), 21–39.
- Roberts, M. J. and Schlenker, W. (2010). *Identifying Supply and Demand Elasticities of Agricultural Commodities: Implications for the US Ethanol Mandate*. Working Paper 15921, National Bureau of Economic Research.
- Scheinkman, J. A. and Schechtman, J. (1983). A simple competitive model with production and storage. *The Review of Economic Studies*, 50(3), 427–441.
- Seale, J. L., Jr. and Regmi, A. (2006). Modeling international consumption patterns. *Review of Income and Wealth*, 52(4), 603–624.
- Skoufias, E., Davis, B. and de la Vega, S. (2001). Targeting the poor in Mexico: An evaluation of the selection of households into PROGRESA. *World Development*, 29(10), 1769–1784.
- Slesnick, D. T. (1998). Empirical approaches to the measurement of welfare. *Journal of Economic Literature*, 36(4), 2108–2165.
- Stennek, J. (1999). The expected consumer’s surplus as a welfare measure. *Journal of Public Economics*, 73(2), 265–288.

- Turnovsky, S. J. (1976). The distribution of welfare gains from price stabilization: The case of multiplicative disturbances. *International Economic Review*, 17(1), 133–148.
- Turnovsky, S. J., Shalit, H. and Schmitz, A. (1980). Consumer’s surplus, price instability, and consumer welfare. *Econometrica*, 48(1), 135–152.
- Vartia, Y. O. (1983). Efficient methods of measuring welfare change and compensated income in terms of ordinary demand functions. *Econometrica*, 51(1), 79–98.
- Waugh, F. V. (1944). Does the consumer benefit from price instability? *The Quarterly Journal of Economics*, 58(4), 602–614.
- Williams, J. C. and Wright, B. D. (1991). *Storage and Commodity Markets*. New York: Cambridge University Press.
- Wright, B. D. (1979). The effects of ideal production stabilization: A welfare analysis under rational behavior. *The Journal of Political Economy*, 87(5), 1011–1033.
- Wright, B. D. (2001). Storage and price stabilization. In B. L. Gardner and G. C. Rausser (eds.) *Marketing, Distribution and Consumers*, volume 1B, part 2 of *Handbook of Agricultural Economics*, chapter 14, (pp. 817–861). Amsterdam: Elsevier.
- Wright, B. D. and Williams, J. C. (1982a). The economic role of commodity storage. *The Economic Journal*, 92(367), 596–614.
- Wright, B. D. and Williams, J. C. (1982b). The roles of public and private storage in managing oil import disruptions. *The Bell Journal of Economics*, 13(2), 341–353.
- Wright, B. D. and Williams, J. C. (1988a). The incidence of market-stabilising price support schemes. *The Economic Journal*, 98(393), 1183–1198.
- Wright, B. D. and Williams, J. C. (1988b). Measurement of consumer gains from market stabilization. *American Journal of Agricultural Economics*, 70(3), 616–627.