Rules versus discretion in food storage policies*

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Abstract

This paper compares different policies in a rational expectations storage model with risk-averse consumers and incomplete markets calibrated to represent a self-sufficient developing country. We consider two state-contingent optimal policies, under commitment and under discretion, and two optimal simple rules of storage, a constant private storage subsidy and a price-band with capacity constraint on public storage. The four policies are designed to be welfare maximising. Commitment allows government to manipulate producers' expectations and induce them to stabilise prices to complement the effect of public storage. Simple rules of stabilisation, more easily implementable than state-contingent policies, when designed optimally can achieve more than two-thirds of the maximum gain. The price-band maximising social welfare is a price-peg scheme: the floor and ceiling prices are the same, and the capacity constraint represents 11% of the steady-state production level.

Keywords: commitment, discretion, food price stabilisation, incomplete markets, storage.

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1 Introduction

Should food prices in poor countries be stabilised? This question was prompted by the riots during the 2007–08 food crisis and, in its aftermath, most commentators agreed about the need for government intervention. The precise design of this intervention, however, is achieving much less consensus. Von Braun and Torero (2009) propose the establishment of a small global food reserve for emergency intervention, accompanied by a virtual reserve designed to reduce irrational speculation on futures markets. Wright (forthcoming) points to the lack of precise motivation for this type of financial scheme. This recalls a similar policy implemented by the US in the 1930s, which proved very expensive and was rapidly dismantled. Various storage schemes have been tried in the past in the bid to stabilise commodity prices, many of which were not motivated by food security. There have been international commodity agreements that relied on price-bands (Gilbert, 1996); in Latin America and the US, private storage subsidies, often in the form of interest-rate subsidies, were common (Gardner and López, 1996); India and the European Union introduced minimum support prices for cereals, but, because stock selling provisions were either unclear or ineffective, in both cases episodes of excessive stock levels occurred (Gardner, 1996, Dorosh, 2009).

Here, we compare storage policies designed to stabilise food prices in a poor, self-sufficient country. Public storage policies are introduced into a rational expectations storage model, in order to stabilise markets beyond the efforts made by private agents. In traditional storage models, stabilisation policies cannot increase welfare since there is no market imperfection (Scheinkman and Schechtman, 1983). In our model, the additional stability brought by storage policies improves welfare because markets are assumed to be incomplete and consumers are risk averse. The use of a rational expectations storage model is motivated empirically by Cafiero et al. (2011), who show that this model performs well for explaining commodity price behaviour. As the present analysis adopts a theoretical approach, we do not try to reproduce the dynamics of a specific market, but rather calibrate the model to values typical of the cereals market in a developing country.

This article characterises fully optimal storage rules, which are nonlinear functions of state variables, and also optimal simple rules, which are simpler policies defined by a limited set of parameters. For simple rules, we consider a constant subsidy to private storage and a price-band with capacity constraint on public stock levels. When government is able to commit to following an optimal state-contingent storage rule, it can induce producers to promote stabilisation by manipulating their expectations, which explains that a policy with commitment generates the highest welfare gains. The gains from commitment, however, are small, and a discretionary policy achieves similar gains. If the parameters of simple rules of stabilisation are chosen so as to maximise welfare, such rules can achieve more than two-thirds of the maximum gains.

The problems created by discretionary intervention in food markets are referred to frequently in the food policy literature (e.g., Poulton et al., 2006, Tschirley and Jayne, 2010). In Africa, private agents cannot easily anticipate discretionary public interventions and so they limit their investments, which increases price instability. To identify the best way to intervene while accounting for the expectations and behaviour of private agents, we use an approach that follows the literature on optimal dynamic policies. The optimal state-contingent policies are derived using the methods in Marcet and Marimon (1999) and Klein et al. (2008). The source of time-inconsistency in the optimal policy is identified and discussed (Kydland and Prescott, 1977). Comparison of fully optimal policies and optimal simple rules is influenced by related works in the theory of monetary and fiscal policies (Schmitt-Grohé and Uribe, 2007, Taylor and Williams, 2010), where simple rules are seen as a type of policy that is more easily implementable and more robust than a fully optimal policy.

This work is related to studies of commodity price stabilisation schemes, for example, Miranda and Helmberger (1988) and Wright and Williams (1988). The former analyse the behaviour of price-band programmes and the latter compare several stabilisation schemes applied frequently in agricultural policies. Both studies emphasise the importance of distinguishing between short-run and long-run effects. Glauber et al. (1989) analyse the cost-efficiency of various policies, and show that a programme of direct payments is the most efficient farm prices stabiliser. Gardner and López (1996) compare policies aimed at stimulating private stockpiling showing that interest-rate subsidies are inefficient and that it is less costly to increase stabilisation through direct storage subsidy. We draw on this literature although our approach differs in that in our work the storage policies, or the parameters that define them, are optimal, while previous studies generally impose exogenous policy rules. Previous studies also do not introduce rationales for public intervention, thus in their framework public interventions are bound to decrease welfare.

Some authors tried to combine stabilisation policies with market imperfections, using the latter as justification for the former. Brennan (2003) uses the limited development of financial markets, which implies prohibitively high private storage costs, to justify public intervention in the rice market in Bangladesh. She proposes stabilisation policies to increase welfare, but does not design them to be welfare maximising. A few works propose stabilisation policies resulting from government optimisation in reaction to market imperfections. Gardner (1979) considers that periods of high prices generate external costs not accounted for by private storers. He proposes optimal storage policies that correct for these external costs. Wright and Williams (1982) design optimal public storage rules where government is unable to commit to not using a price ceiling, which creates a disincentive for private storage.[1] Although we build on these works, we extend the analysis of optimal policies by distinguishing between commitment and discretionary policies. We consider also the outcomes of suboptimal policies, such as storage subsidy or price-bands, which figure more frequently in policy discussion than fully optimal rules.

Section 2 describes the rational expectations storage model without public intervention. Section 3 introduces a framework for designing optimal policies. In Section 4, we describe the calibration of the model and discuss the numerical results. Section 5 concludes.

[1] See also McLaren (1997, 1998) for interesting discussions on optimal policies.

2 The model

Our analysis follows usual practice in studies of commodity price stabilisation schemes (Miranda and Helmberger, 1988, Wright and Williams, 1988) and uses a rational expectations storage model. This is a partial equilibrium model featuring a market for a storable commodity with a competitive storer, a producer whose output is submitted to multiplicative shocks, and a final demand. We assume that consumers are risk averse and unable to insure because of market incompleteness. In our model, time is discrete, and one period is assumed to correspond to one year. Thus, we focus on inter-annual storage and ignore issues related to seasonal fluctuations.

2.1 Consumers

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification: $D(P_t, Y) = dP_t^{\alpha}Y^{\eta}$, where $d > 0$ is a normalisation parameter; P_t is the period t price; Y is income, which is assumed to be constant; and α, with $\alpha < 0$ and $\alpha \neq -1$, and $\eta \neq 1$ are the price and income elasticities. Assuming there are only two goods and the second good is the numeraire, the integration of this demand function gives the following instantaneous indirect utility function (Hausman, 1981)

$$\hat{v}(P_t, Y) = \frac{Y^{1-\eta}}{1-\eta} - d\frac{P_t^{1+\alpha}}{1+\alpha}. \tag{1}$$

This utility function has a relative risk aversion equal to the income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a): we assume $\hat{v}(P_t, Y)$ to be positive and apply a monotone transformation to the indirect utility function,

$$v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1+\theta}. \tag{2}$$

with $v(P_t, Y) \to \ln \hat{v}(P_t, Y)$ as $\theta \to -1$. This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is

$$\rho(P_t, Y) = \eta - \theta\frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}, \tag{3}$$

with θ indexing the degree of risk aversion.

For the sake of simplicity, the representative consumer is assumed to adopt a hand-to-mouth behaviour: he consumes current income and does not save to smooth out fluctuations. The dynamics are thus simplified, since consumer's "cash on hand" does not have to be included as a state variable. This assumption overlooks the role of self-insurance through saving. However, such self-insurance remains limited in practice and falls short of providing protection comparable to what complete markets deliver, due *inter alia* to borrowing constraints and to the rather large budget share of staple food in many developing countries.

2.2 Storers

There is a single representative speculative storer, which is risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity S_t^{s} from period t to period $t+1$ entails a purchasing cost, $P_t S_t^{\mathrm{s}}$, and a storage cost, $k S_t^{\mathrm{s}}$, with k the unit physical cost of storage. We introduce also a per-unit constant subsidy ζ, which is considered later as a possible stabilisation tool. The benefits in period t are the proceeds from the sale of previous stocks: $P_t S_{t-1}^{\mathrm{s}}$. The storer follows a storage rule that maximises its expected profit defined by

$$V^S\left(S_{t-1}^{\mathrm{s}}, P_t\right) = \max_{\left\{S_{t+i}^{\mathrm{s}} \geq 0\right\}_{i=0}^{\infty}} \mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[P_{t+i} S_{t+i-1}^{\mathrm{s}} - \left(P_{t+i} + k - \zeta\right) S_{t+i}^{\mathrm{s}} \right] \right\}, \tag{4}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, and β is the discount factor. The storer's problem can be expressed in a recursive form using the following Bellman equation:

$$V^S\left(S_{t-1}^{\mathrm{s}}, P_t\right) = \max_{S_t^{\mathrm{s}} \geq 0} \left\{ P_t S_{t-1}^{\mathrm{s}} - \left(P_t + k - \zeta\right) S_t^{\mathrm{s}} + \beta\, \mathrm{E}_t \left[V^S\left(S_t^{\mathrm{s}}, P_{t+1}\right) \right] \right\}. \tag{5}$$

This equation has two state variables: the price, whose dynamics is considered by the storer to be exogenous, and the stock carried over from the previous year. Using the first-order condition on S_t^{s} and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition[2]

$$S_t^{\mathrm{s}} \geq 0 \quad \perp \quad \beta\, \mathrm{E}_t\left(P_{t+1}\right) + \zeta - P_t - k \leq 0, \tag{6}$$

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. So this is a situation of a stabilising speculation, the storer buys when prices are low and when rationally it expects that they will become higher later.

2.3 Producers

A representative producer makes his productive choice one period before bringing output to market. He puts in production in period t a level H_t for period $t+1$, but a multiplicative disturbance (e.g., a weather disturbance) affects final production. The producer chooses the production level by solving the following maximisation of expected profit:

$$\max_{\left\{H_{t+i}\right\}_{i=0}^{\infty}} \mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[P_{t+i} \epsilon_{t+i} H_{t+i-1} - \Psi\left(H_{t+i}\right) \right] \right\}, \tag{7}$$

[2] Complementarity conditions in what follows are written using the "perp" notation ($\perp$). This means that the expressions on either side of the sign are orthogonal. If one equation holds with strict inequality, the other must have an equality.

where $\Psi(H_t)$ is the cost of planning the production H_t and $H_t\epsilon_{t+1}$ is the realised level. ϵ_{t+1} is the realisation of an i.i.d. stochastic process of mean 1 exogenous to the producer, which follows a translated beta distribution. Like the storer's problem, this problem can be reformulated into a recursive form, and its solution gives the following Euler equation:

$$\beta \,\mathrm{E}_t\left(P_{t+1}\epsilon_{t+1}\right) = \Psi'\left(H_t\right). \tag{8}$$

This equation has a straightforward interpretation: it is the equality between the marginal cost of production and the expected discounted marginal benefit of one unit of planned production. We assume decreasing returns to scale since a production increase requires the use of less fertile lands. The production cost function follows an isoelastic form

$$\Psi\left(H\right) = h\frac{H^{1+\mu}}{1+\mu}, \tag{9}$$

where $h > 0$ is a scale parameter and $\mu \geq 0$ is the inverse of supply elasticity.

2.4 Recursive equilibrium

At the beginning of each period, three predetermined variables define the state of the model: S^{S}_{t-1}, H_{t-1} and ϵ_t. They can be combined in one state variable, private availability, the sum of production and private carry-over:

$$A^{\mathrm{P}}_t = S^{\mathrm{S}}_{t-1} + H_{t-1}\epsilon_t. \tag{10}$$

When government pursues food price stabilisation through public storage, we can also define total availability as the sum of private availability and public carry-over noted S^{G}:

$$A^{\mathrm{T}}_t = S^{\mathrm{G}}_{t-1} + S^{\mathrm{S}}_{t-1} + H_{t-1}\epsilon_t. \tag{11}$$

Accounting for the presence of public stockholding, market equilibrium can be written as[3]

$$A^{\mathrm{T}}_t = A^{\mathrm{P}}_t + S^{\mathrm{G}}_{t-1} = D\left(P_t\right) + S^{\mathrm{S}}_t + S^{\mathrm{G}}_t. \tag{12}$$

From the above, we can define the recursive equilibrium of the problem without public intervention:

Definition. In the absence of a stabilisation policy (i.e., $\zeta = S^{\mathrm{G}} = 0$), a recursive equilibrium is a set of functions, $S^{\mathrm{S}}\left(A^{\mathrm{P}}\right)$, $H\left(A^{\mathrm{P}}\right)$ and $P\left(A^{\mathrm{P}}\right)$, defining storage, production and price over the state A^{P} and a transition (10) such that (i) storer solves (4), (ii) producer solves (7), and (iii) the market clears.

[3] As income is assumed to be constant, in what follows the demand function is expressed only as a function of price.

3 Designing optimal stabilisation policies

The monetary policy literature distinguishes two types of policies (Clarida et al., 1999, Taylor and Williams, 2010). State-contingent policies, designed to be fully optimal in a given model, which serve as theoretical benchmarks, and simple rules or policies defined with few parameters, which are considered more practical since they are less model-dependent and are simpler to communicate and implement. We follow this logic and examine both optimal state-contingent storage policies under commitment and under discretion, and simple rules. State-contingent rules are designed using the methodology developed in Marcet and Marimon (1999) for commitment, and in Klein et al. (2008) for discretion.

Policies start at period 0 and are unanticipated by the agents. The parameters of stabilisation policies are determined by maximising a social welfare function that aggregates consumers' utility, other agents' surpluses and fiscal cost. Initial availability is taken as the deterministic steady-state level and initial private stocks are assumed to be null. Government is assumed to be able to manage storage facilities at the same costs as private storers.

3.1 Social welfare function

In a partial equilibrium model, it is not possible to take the expected sum of discounted consumer utility as the policy authority objective as is usual in optimal policy problems. Instead, the welfare of all the agents must be included in the objective. The approach in this article also differs from most partial equilibrium analyses in considering risk aversion. In partial equilibrium models, policy design is based on maximising the sum of all agents' surpluses. This takes no account of risk aversion, since the expected consumer surplus is not a measure of the welfare of risk-averse agents (Helms, 1985b). Instead of using the sum of agents' surpluses, we combine the utility of consumers with the surplus of other agents.

Thus, we introduce a social welfare function, which weights linearly the welfare of each agent. At period t, social welfare is given by

$$W_t = v\left(P_t, Y\right) + w\left[P_t H_{t-1}\epsilon_t - \Psi\left(H_t\right) + P_t S^{\text{s}}_{t-1} - \left(P_t + k - \zeta\right) S^{\text{s}}_t - Cost_t\right], \tag{13}$$

where w is the weight in social welfare to monetary terms, and $Cost_t$ is the fiscal cost of the policy. The use of a unique weight for all monetary terms implies implicitly that any distortionary cost caused by tax revenue collection is neglected.

The weight w is chosen such that it ensures that transfers (i.e., changes in average expenditures/profits/costs) between agents do not affect welfare. Storage policies affect price distributions, which enables the desired risk sharing. In addition to spreading risk among agents the changes in price distributions create monetary transfers. In order to focus on efficiency issues, we assume that the level of social welfare cannot be improved by these transfers and choose the weight accordingly. To rule out transfers, w is defined as the value in utility terms of a unitary monetary transfer to consumers. The benchmark situation we consider to determine w is the ergodic distribution of the model without public intervention. Given this definition, a small, permanent, monetary transfer, δ, to consumers should not change welfare:

$$\mathrm{E}\left[v\left(P, Y+\delta\right)-w\delta\right]=\mathrm{E}\left[v\left(P, Y\right)\right]. \tag{14}$$

A first-order approximation of the left-hand side around $\delta = 0$ gives w equal to unconditional expectation of the marginal utility over income:

$$w=\mathrm{E}\left[v_Y\left(P, Y\right)\right]. \tag{15}$$

The cost of the policy is either the amount of the subsidies given to private storers or the cost of carrying public stock, which is similar to the profit from private storage. It is equal to the difference between purchasing plus storage costs of new stock, and the revenue derived from selling previous period stock, S_{t-1}^{G}:

$$Cost_t=\zeta S_t^{\mathrm{S}}+\left(P_t+k\right)S_t^{\mathrm{G}}-P_t S_{t-1}^{\mathrm{G}}. \tag{16}$$

Using equations (10) and (16), social welfare can be simplified as

$$W_t=v\left(P_t, Y\right)+w\left[P_t\left(A_t^{\mathrm{P}}+S_{t-1}^{\mathrm{G}}\right)-\Psi\left(H_t\right)-\left(P_t+k\right)\left(S_t^{\mathrm{S}}+S_t^{\mathrm{G}}\right)\right]. \tag{17}$$

In the social welfare function, the subsidy is positive in storers' profit and negative in public cost, so eventually it disappears.

3.2 State-contingent policies

State-contingent policies are public storage policies in which storage decisions are based on the state of the system. The state of the system is defined by total availability. We consider two state-contingent policies based on government's ability to commit to a policy rule. In both cases, government maximises the expected intertemporal social welfare function. If it is able to commit to an intervention rule, its storage rule is defined at the first period of the policy implementation and applied for all the following periods, while without commitment, government chooses the optimal storage level at each period. For conciseness, the dynamic programming problem and first-order conditions of the optimal policy problems are presented in the appendix (for a detailed interpretation of similar first-order conditions, see Gouel, 2010).

3.2.1 Policy under discretion

For the discretionary policy, we consider a time-consistent Markovian policy (see, e.g., Klein et al., 2008, and Ambler and Pelgrin, 2010, for characterisation of the equilibrium concept). The government is, in each period, a Stackelberg leader, moving first while accounting for agents' best response to the policy (i.e., their first-order conditions). The choice concerns current-period endogenous variables and depends only on the current state, total availability, and the value of future variables is taken as given based on rational expectations about them. So, at each period, the policy maker maximises the conditional expected sum of the discounted social welfare function subject to the equations defining the rational expectations equilibrium:

$$\max_{\substack{S_t^{\mathrm{S}}\geq 0, H_t, P_t,\\ A_{t+1}^{\mathrm{T}}, S_t^{\mathrm{G}}\geq 0}} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ v\left(P_{t+i}, Y\right) + w\left[P_{t+i} A_{t+i}^{\mathrm{T}} - \Psi\left(H_{t+i}\right) - \left(P_{t+i} + k\right)\left(S_{t+i}^{\mathrm{S}} + S_{t+i}^{\mathrm{G}}\right)\right]\right\} \tag{18}$$

subject to equations (6), (8), (11) and (12), A_t^{T} given, and anticipating $\left\{S_{t+i}^{\mathrm{S}}, H_{t+i}, P_{t+i}, A_{t+i+1}^{\mathrm{T}}, S_{t+i}^{\mathrm{G}}\right\}$ for $i \geq 1$.

3.2.2 Policy under commitment

The optimal policy rule under commitment is set at the initial period and maintained unchanged for all subsequent periods. It is defined to maximise intertemporal social welfare, and so solves the following problem

$$\max_{\left\{\substack{S_t^{\mathrm{S}}\geq 0, H_t, P_t,\\ A_{t+1}^{\mathrm{T}}, S_t^{\mathrm{G}}\geq 0}\right\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ v\left(P_t, Y\right) + w\left[P_t A_t^{\mathrm{T}} - \Psi\left(H_t\right) - \left(P_t + k\right)\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}}\right)\right]\right\} \tag{19}$$

subject to equations (6), (8), (11) and (12), and A_0^{T} given.

One difference between discretion and commitment is that under the latter, the storage rule is history-dependent: the storage level decided at one period depends on behaviour in previous periods. This dependence is created by the commitment. By committing to a rule government is able to manipulate the expectations of private agents. In a rational expectations equilibrium, government's actions confirm private agents earlier expectations. So this manipulation of expectations comes at the cost of the need to respect past promises, which creates path-dependence (formally, these promises are represented in the first-order condition (30) in the appendix by lagged Lagrange multipliers, which are introduced as additional state variables).

3.3 Optimal simple rules

Simple rules are rules of public behaviour providing a simple feedback between observable variables and policy instruments. To be simple, these rules have to be defined by a small set of parameters, and not be infinite-dimension objects like the state-contingent policies defined above. We consider two simple rules commonly discussed in the literature: a constant subsidy for storage, which aims at stimulating private storers' activity, and a price-band defended by a public stock. They are defined respectively by one and three parameters, which are chosen to maximise intertemporal social welfare.

Since these rules are not fully flexible with respect to the state of the system, in contrast to a state-contingent policy, they do not provide an optimal policy for each situation. Despite this limited flexibility, simple rules are discussed at length in the literature on monetary and fiscal policy, which recognises their good properties. Simple rules entail commitment. They also tend to be more robust than globally optimal policies. The need for robustness emerges from our limited knowledge of the way the economy works. The literature does not completely agree about the best explanations for agricultural price dynamics; and it might be better to find a simple rule that is robust across a variety of frameworks than to design optimal policies that are model-specific and whose good properties do not transfer well to another model (Levin et al., 2003). Gardner (1979) analyses this issue of robustness in a situation where ignorance is related to the level of negative externality created by high commodity prices. He shows, in that context, that a price-band policy may be preferable to an optimal stockpiling policy. In the present study, we do not analyse the robustness of simple rules. We only consider, in a given framework, how simple rules compare with optimal state-contingent rules.

Unlike state-contingent rules, the parameters of optimal simple rules are not determined by solving the first-order conditions of a maximisation problem. The parameters that need to be determined are constant, and maximising over constant variables creates terms with infinite sums that are difficult to handle. To find these parameters, a method similar to the nested fixed-point approach in Rust (1987) is applied. It consists of two nested algorithms. The inner one solves the rational expectations problem for a given set of policy parameters. The outer one adjusts the parameters of the rule to maximise intertemporal social welfare by applying an optimisation solver.

3.3.1 Subsidy to storage

A constant subsidy to storage is a very simple policy, since it consists only of giving private storers a constant subsidy ζ for each stockpiled unit. The intervention does not vary, there is no feedback between observations and policy actions, the underlying idea being that private storers do a good job at reacting to the economic situation, but provide too little stabilisation and should be encouraged to do more. The subsidy makes physical speculation more profitable and increases stock levels, which stabilises prices more than in the absence of subsidy. Since there is no public storage, storage subsidy makes private availability equal to total availability.

The optimal subsidy is the value maximising the conditional expected sum of the discounted social welfare, defined by (17), subject to (6), (8), (10) and (12).

3.3.2 Price-band

A price-band programme is a controversial policy. On the one hand, it is probably the most commonly proposed stabilisation instrument in the policy arena. Its simplicity is appealing to policy makers. It consists of defining two prices: a floor and a ceiling price, which would be defended by a public stock. It is a function of current price, which is, in practice, more easily observable than the availability on market. Such a scheme was proposed in 1942 by Keynes (1974) as a way to stabilise internationally traded raw materials. Subsequently, it was used in three international commodity agreements: cocoa, rubber and tin (Gilbert, 1996). BULOG, the Indonesian Food Logistics Agency, has defended, with some success in the rice market, a floor price to protect farmers and a ceiling price to protect consumers (Timmer, 1997).

On the other hand, theoretical analysis of price-bands has led to much criticism (Miranda and Helmberger, 1988, Williams and Wright, 1991, Ch. 13–14). One major critique of price-band programmes is that they tend to over-accumulate stock since nothing is sold until the ceiling price is reached. They may also lead to explosive stock levels in the case of poor combinations of floor and ceiling prices. Finally, policy makers expect these programmes to stabilise prices between the bounds, but the models predict that, on the contrary, prices tend to get stuck at the bounds, challenging them continually, but will rarely be lying between them.[4]

In our implementation of a price-band, to avoid extreme over-accumulation, we place a capacity constraint, $\bar{S}^{\mathrm{G}}$, on public storage.[5] Once public stock reaches its capacity constraint, public intervention ceases and the floor price is no longer defended until there is a decrease in the public stock. So this policy is governed by three parameters: a floor price, P^{F}, a ceiling price, P^{C}, and a capacity constraint. The parameters are determined optimally to maximise welfare, so if the capacity constraint does not improve welfare, it will be given a very high value thus making it redundant.

The behaviour of a price-band with capacity constraint obeys some simple principles. When the price is above the floor price, there is no accumulation of public stock:

$$P_t > P^{\mathrm{F}} \Rightarrow \Delta S_t^{\mathrm{G}} \leq 0. \tag{20}$$

Likewise, when the price is below the ceiling price, government does not sell stock:

$$P_t < P^{\mathrm{C}} \Rightarrow \Delta S_t^{\mathrm{G}} \geq 0. \tag{21}$$

When the capacity constraint is reached, the floor price is not defended and the price can decrease below it:

$$P_t < P^{\mathrm{F}} \Rightarrow \Delta S_t^{\mathrm{G}} = \bar{S}^{\mathrm{G}} - S_{t-1}^{\mathrm{G}}. \tag{22}$$

Finally, the ceiling price is not defended when the public stock is exhausted:

$$P_t > P^{\mathrm{C}} \Rightarrow \Delta S_t^{\mathrm{G}} = -S_{t-1}^{\mathrm{G}}. \tag{23}$$

These four conditions are expressed in a concise mathematical formulation as two mixed complementarity equations in the appendix.

Under a price-band policy, the problem has two state variables: private availability, defined by (10),

[4] For a non-technical summary of all criticisms, see Wright (forthcoming).

[5] Alternatively, a maximum public spending could have been defined. However, a storage capacity constraint is simpler to express mathematically.

and previous period public stock. Public stock appears as a separate state variable, because it is not directly available for the market. It is used only to defend the price-band and so does not play the same role as private stock and production, which can be summed together.

4 Results

The rational expectations storage model is known to lack a closed-form solution. It has to be approximated numerically. The numerical method used here is inspired by Fackler (2005) and Miranda and Glauber (1995). It is a projection method with a collocation approach. A grid on the state variables is chosen, on which decision rules are approximated by splines. Equilibrium equations are solved by the mixed complementarity problems solver `PATH` (Dirkse and Ferris, 1995).[6] The number of grid points is chosen such that the use of these decision rules entails less than a $1 error on average for every $1,000 of decision (measured by the Euler equation error).

4.1 Calibration

Table 1 presents the parameters used to calibrate the model. The parameters are set such that, at the model's non-stochastic steady state, price, production, consumption and availability are equal to 1. An annual interest rate of 5% is used for discounting.

Table 1. Parameterisation

Parameter	Economic interpretation	Assigned value
β	Annual discount factor	0.95
η	Income elasticity	0.5
α	Own-price demand elasticity	-0.4
γ	Commodity budget share	0.15
μ	Inverse of supply elasticity	2
Y	Income	6.67
d	Normalisation parameter of demand function	0.39
h	Normalisation parameter of production cost function	0.95
θ	Parameter defining risk aversion	-2.63
k	Physical storage cost	0.06
ϵ	Probability distribution of yield	$B(2,2)\cdot 0.5+0.75$

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: -0.4 for price elasticity and 0.5 for income elasticity. We assume that consumers spend, at the steady state, $\gamma = 15\%$ of their income on the staple (a value intermediate between what is observed for rice consumption in poor and affluent households in Asia, Asian Development Bank, 2008). Since steady-state consumption

[6]For more precisions, see the `RECS` solver (`https://github.com/christophe-gouel/recs`), with which the model is solved.

and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, $1/\gamma$. We assume at the steady state a relative risk aversion parameter of 2, implying $\theta = -2.63$.

We follow Brennan (2003) and assume a per-unit storage cost of 6% of the steady-state price (i.e., $k = 0.06$). Combine with opportunity cost, this physical storage cost entails an overall storage cost at steady state equal to 11.3% of the steady-state price.

Estimation of the price elasticity of supply produces controversial results (Rao, 1989, Schiff and Montenegro, 1997). We choose a value of 0.5, often produced by related studies, and analyse later the sensitivity of results to this value. For this elasticity, we have $\mu = 2$. The productive shocks, ϵ, are assumed to follow a beta distribution of shape parameters, 2 and 2, which makes it unimodal at 0.5 and symmetric. The distribution is recentredrecentered and rescaled to vary between 0.75 and 1.25, which implies a coefficient of variation of 11.2%.

4.2 Storage behaviour

Without storage policy, private storers (solid line in figures 1–2) do not stock anything for low availability (the threshold is close to the steady-state availability level, 1), and increase the level of stock with market availability above the threshold. When normal consumption is satisfied, any additional quantity in the market tends to lower prices. The speculators take opportunity of these lower prices to accumulate stock that can be sold in periods of lower availability. If government intervenes, total stock is always higher for a given availability than without intervention.

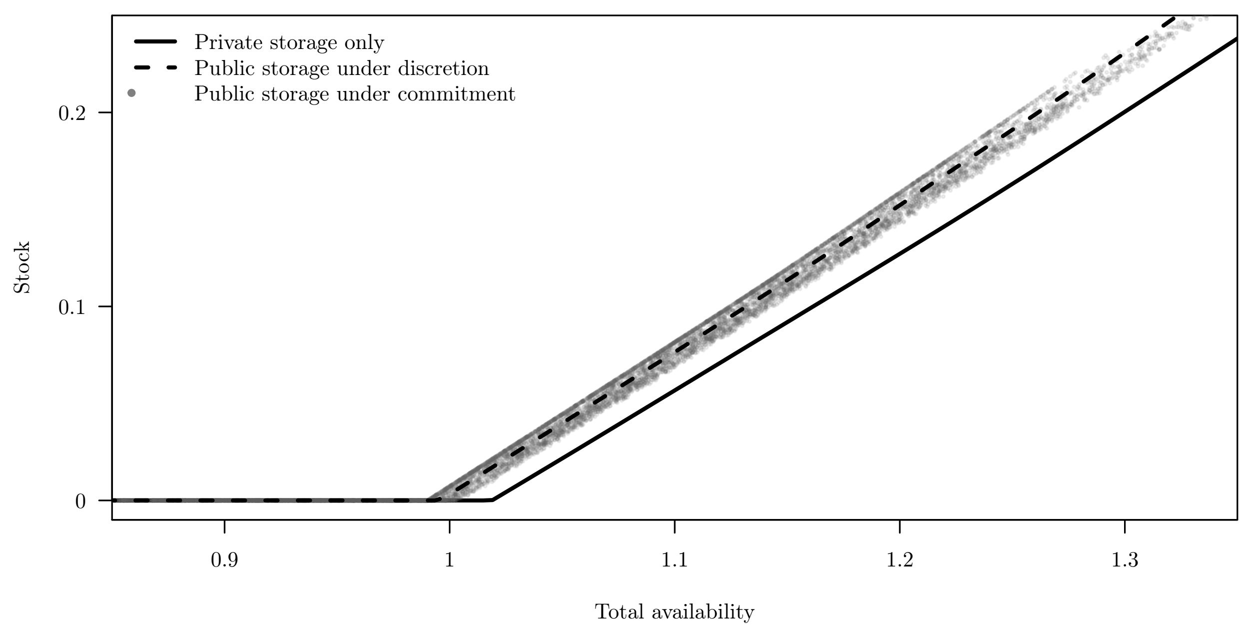

Figure 1. State-contingent storage rules. Grey points are 10,000 simulated points from the asymptotic distribution of the optimal storage policy under commitment.

4.2.1 State-contingent public storage

State-contingent public storage completely crowds out private storage. Public stockpiling starts at a lower total availability than would do private storers, and public storage rules present a higher marginal propensity to store than the private storage rule without intervention. The higher stock level results in prices that are more stable. This stabilisation is such that there is no profit opportunity left for speculation.

Under commitment, the storage rule is not defined only by total availability (see the grey dots in Figure 1, these points are obtained by simulation of the optimal storage rule), which illustrates the history-dependence of the rule. As explained above, history-dependence arises as the need to respect past commitments in a rational expectations model. Since private storers are crowded out, government commitments concern only agents with intertemporal trade-offs: producers.

Path-dependence works as follows. For a given current availability A_t^{T}, the lower the previous availability, A_{t-1}^{T}, the higher will be the current public stock, S_t^{G}, and vice versa. When availability is low, it is difficult to stockpile without hurting consumers by raising prices too much. However, to ensure a future sufficient availability, government promises producers a high price at the next-period, thus creating incentives for high production levels. At the next-period, the only way for government to meet its commitments is to stockpile more in order to raise the price for producers. Conversely, when availability is high, government would like to decrease production more than producers would tend to do so. It thus promises a lower price at the next period and, in order to keep this promise, it has to stockpile less than it would have done without commitment in order to force the price down. Were government to renege on its promise, it would reset the lagged Lagrange multipliers to zero and choose another storage level. For most levels of availability, the level of storage under commitment may be below or above the level under discretion depending on the state's history. However, for availabilities close to or above 1.3, the storage rule under commitment is always below the rule under discretion. A situation with such high availability levels can only follow another period of high availability, hence it implies public engagement to limit stockpiling in order to decrease the expected price and limit production.

Public storage under discretion (short-dash line) is close to the level under commitment, one main difference being that it is a function only of current availability and is not time-dependent. Under discretion, government cannot commit to a rule and so has no need to respect past promises.

4.2.2 Subsidy to private storage

A policy of subsidy to private storage shifts the storage rule to the left (black, long-dash curve in Figure 2). It produces a storage rule that is close to the optimal, state-contingent, discretionary rule. The optimal level of subsidy is 0.04, which represents 35% of steady-state storage cost (physical cost plus opportunity cost for a price of 1).

We have assumed a constant subsidy. This is convenient for policy making since this policy is simple and implies commitment. The drawback is that it lacks the flexibility of a state-contingent public

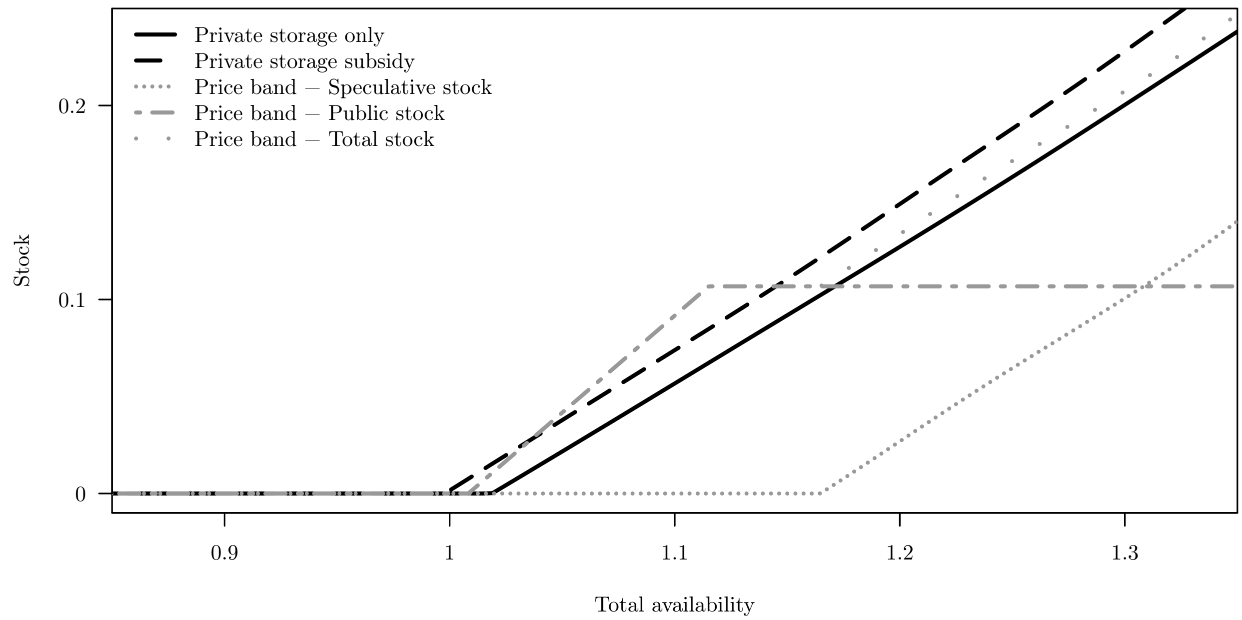

Figure 2. Storage rules for optimal simple rules policies

storage to adjust to a changing situation. Other schemes of subsidies could have been considered. For example, instead of being constant a subsidy to private storage could be state-contingent. A state-contingent subsidy allows decentralisation of an optimal public storage policy by compensating the speculator for its loss when following a welfare maximising storage-rule.

4.2.3 Price-band programme

When the price-band policy is designed to maximise welfare, the optimal floor and ceiling prices collapse in a single price, equal to 0.98. This kind of policy, a floor-price with a capacity constraint on the public stockpile, is analysed in Salant (1983), Williams and Wright (1991, Ch. 14) and Brennan (2003), and dubbed a "price-peg", the term we use in what follows. Government defends the target price by building stock, but when price is above target, public stock is sold. This behaviour is as is expected under a price-band except that the two bounds are confounded. This result contrasts with the intuition that to protect consumers, stocks should be accumulated at low prices and used to prevent prices from exceeding a limit. However, this is in line with the theory on price-band programmes. Miranda and Helmberger (1988, table 3) show that the deadweight loss from a price-band programme is the lowest for a zero bandwidth. Williams and Wright (1991, Ch. 14) also find that the price-peg scheme entails smaller losses than either a price-band or a simple floor-price. They also explain that a major flaw in price-band schemes is that they tend to over-accumulate. This is caused by the absence of public stock sales while the price is between the bounds. A price-band policy does not try to stabilise the price inside the bounds, but only to prevent the price from exceeding them. Under a price-band scheme, public stock is not considered as a potential source of supply, but only as a mean of defending the bounds, so the stock can reach very high levels if the bounds are set inappropriately. A price-peg scheme avoids this drawback by putting stock on the market as soon as it is accumulated. Excessive accumulation is also avoided by the existence of a capacity limit.

The optimal coincidence of floor and ceiling prices may be sensitive to one assumption rarely considered in the storage literature, but likely relevant: the transaction costs involved in adjusting stock levels (Chavas et al., 2000). For example, for storage in a silo, grain has to be cleaned and the temperature reduced to prevent pest infestation, which entails costs. These adjustment costs may justify a small wedge between the two prices to avoid too many stock level adjustments.

Under this price-peg programme, speculators still find profit opportunities (see the grey curves in Figure 2), when public storage is limited by its capacity constraint.[7] For high availability, government cannot defend the price-peg, because its stock bumps into its capacity constraint, optimally set at 0.11. At the threshold point where the capacity constraint becomes binding, the tendency of over-accumulation appears clearly. The storage rule under price-peg scheme is above all other rules and the marginal propensity to store is 1. When the capacity constraint is reached, there is a situation where, even if availability increases, no stock, public or private, is added to the existing stock. Private storers start to accumulate for higher levels of availability.

This interaction between private speculation and public stock is illustrated in the simulation in Figure 3. Public stock frequently reaches its capacity limit. This implies that the price-peg programme does not work as expected for a traditional price-band. The target price is not seriously defended since public stock is often at its limit. By its nature, a price-peg cannot be defended indefinitely if the initial public stock is finite. And, without the capacity constraint, the accumulation of public stock under this policy could be explosive given the high level of the intervention price. Actually, the price-peg is just a way to define a rule allowing the accumulation of public stocks to protect consumers from high prices and from fluctuations, it is not a target that government seriously tries to support.

4.3 Dynamics under food price stabilisation policies

In this section, we analyse the transitional behaviour following policy implementation. We do not analyse the dynamics of endogenous variables for specific simulations, only their expectations while they converge to their new asymptotic distributions. In any period, the realised values may be above or below these expectations. Since the policies involve higher mean stock levels than without intervention, they begin with a phase of stock build-in. The transition is illustrated in Figure 4 with the dynamics of mean total stock and mean prices. The average stock level increases by more than one half under an optimal policy. The lowest stock level increase occurs for the price-peg policy and the highest for the discretionary policy.

[7] For a price-peg defined by other parameters, it is also possible to observe speculators' activity at low availability where they can carry speculative attacks on the public stock (Salant, 1983).

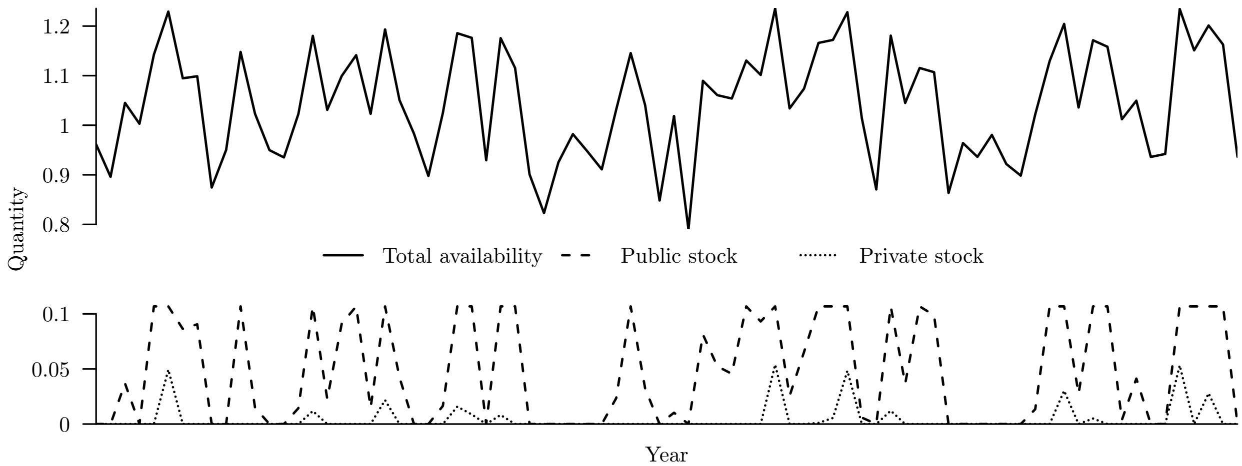

Figure 3. Simulation of total availability, public stock and private stock under the optimal price-band programme

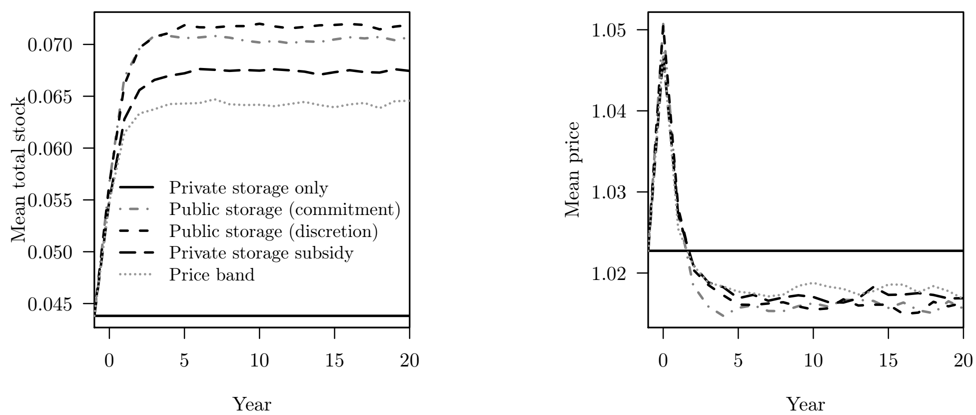

Figure 4. Transitional dynamics. Before period 0, the system is on the asymptotic distribution of the model without public intervention. The unexpected policy is announced and starts in period 0. (Obtained by Monte Carlo simulation as the average over 100,000 simulated paths.)

In the first years, the purchases made to accumulate stocks push mean prices above their long-run values. After a few years, mean prices drop below the value without public intervention. Because of the convexity of the demand curve, the additional storage decreases higher prices more than it increases low prices. Hence, stabilisation entails lower mean prices and the effects are almost identical for all policies.

The dynamic welfare effects of the policies are presented in Table 2. Consumer gains are *ex ante* per-period equivalent variation, EV, calculated using the following implicit definition

$$\mathrm{E}_0\left\{\sum_{t=0}^{+\infty}\beta^t\left[v\left(P_t,Y+EV\right)-v\left(\check{P}_t,Y\right)\right]\right\}=0, \tag{24}$$

where P and $\check{P}$ are the prices before and after stabilisation. Producers gains are the annualised changes in expected profit from equation (7). Government outlays are the annualised expected sum of discounted costs defined by equation (16).[8] Changes in storers' surpluses are ignored since storers operate, in average, at zero profit and we have assumed no stock at the first period. Welfare increases for all policies. As expected, it is maximum for public storage under commitment.

Table 2. Welfare results on transitional dynamics (as a percentage of the steady-state commodity budget share).

	Commitment	Discretion	Subsidy	Price-band
Consumers gains	0.373	0.358	0.325	0.276
Producers gains	0.039	0.028	0.011	−0.007
Government outlays	0.323	0.301	0.253	0.206
Total gains	0.089	0.084	0.083	0.063

Most welfare results differ little across policies. A first conclusion is that there are limited gains from commitment. This is explained by the source of time-inconsistency, which lies in producers' expectations. The manipulation of producers' expectations helps government achieve price stabilisation by giving it an additional instrument: a partial control on planned production, and changing planned production is an alternative to storage to affect next-period availability. However, because supply reaction is small (supply elasticity is 0.5), manipulating producers' expectations entails limited gains (this issue is discussed in more detail in Section 4.5).

An optimal constant subsidy for private storage achieves welfare results close to state-contingent policies. Comparison of the storage rules under discretion and under constant subsidy (figures 1–2) shows that they are fairly similar. In monetary policy theory, a rationale for using simple rules is to achieve most of the gains from commitment via a policy that is easier to implement than a globally optimal policy. The gains from commitment being small here, a constant subsidy does not achieve higher welfare than the policy under discretion. A storage subsidy could still be defended on the grounds that it is a very simple policy to announce and decentralises the policy implementation to private agents.

The price-band, which in our case collapses to a price-peg mechanism, is the worst of the four policies. It manages, nevertheless, to reap more than two-thirds of the welfare gains from the policy under commitment. As shown in Figure 2, one problem with the price-band policy is its tendency to over-accumulate. Because of its marginal propensity to store equal to 1 when the intervention price is defended, stocks accumulate much faster than what is prescribed by the optimal rules. Excessive accumulation is only prevented by the capacity constraint on storage.

The relative good performance of the price-band should be linked to the optimal choice of parameters. Table 3 illustrates what would be the social welfare changes from price-bands with different parameters. It considers three floor prices and three limits on public storage. Ceiling prices are

[8] Welfare terms are all discounted infinite sums. They are calculated by transformation to a recursive formulation and value function iteration.

chosen to illustrate price-pegs, symmetric price-bands and ceilings at the deterministic steady state. There is no monotonic relationship between welfare and any parameter. Increasing one parameter increases welfare for some combinations of the two other parameters and decreases it for other combinations. The capacity constraint is essential for reaching the highest gains; with no constraint or a high one, most parameters' combinations entail welfare losses. Despite the intuitive appeal of defining a policy by a mean price with symmetric bounds, symmetric price-bands do not perform well, since the higher the upper bound, the longer the food is stored. Welfare tends to decrease with the ceiling price, and price-peg schemes achieve the highest welfare levels.

Table 3. Social welfare changes from non-optimal price-band policies (as a percentage of the steady-state commodity budget share).

		Capacity constraint on public stock		
Floor price	Ceiling price	0.15	0.3	$+\infty$
0.85	0.85	0.007	0.019	0.018
0.85	1.00	0.019	−0.003	−0.001
0.85	1.15	−0.053	−0.093	−0.100
0.90	0.90	0.039	0.044	0.038
0.90	1.00	0.031	−0.062	−0.127
0.90	1.10	−0.088	−0.414	−1.001
0.95	0.95	0.060	−0.037	−0.194
0.95	1.00	0.037	−0.191	−0.741
0.95	1.05	−0.019	−0.393	−1.660

Speculative storers are essential for good functioning of the optimal price-band. They stock for prices below the target. Hence they provide some stabilisation in situations where the public storage rule is inactive. Without speculator activity, the price-peg policy would generate welfare losses. Under a price-band programme government intervenes only to defend the bounds and does not consider volatility inside and outside the bounds. Because there are profit opportunities beyond the bounds, speculators bring additional stabilisation.

4.4 The long-run effects of stabilisation policies

On the asymptotic distribution, stabilisation policies decrease the occurrence of low prices (see the shift in the distribution to the right in Figure 5), because there is more storage in situations of high availability than without public intervention. This effect is less important for the price-band policy since public stockpiling starts at higher availability levels.

Price density without public intervention presents a long right tail, where consumers suffer from high prices. All stabilisation policies reduce the frequency of very high prices thanks to the additional storage made at high availability.

For the price-peg policy, the target price is located close to the steady-state price. This means that, without public intervention, prices will often be below this target, implying that under a

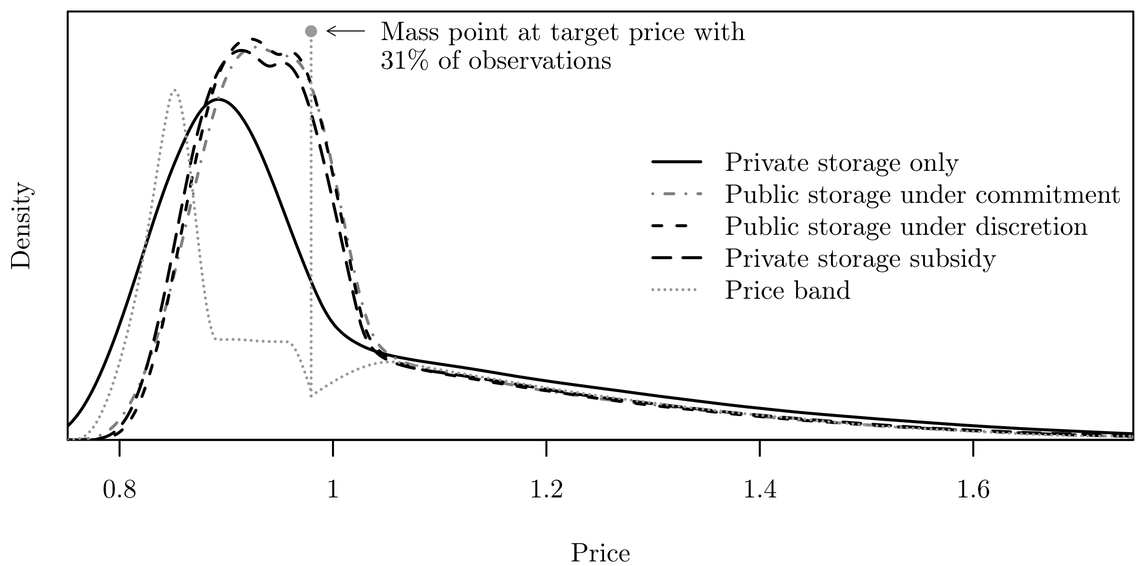

Figure 5. Density of price under various intervention schemes. The curves are obtained by kernel density estimation over 1,000,000 simulated points.

price-peg government intervention will be frequent. We observe that when prices are not on target, they are mostly below it. Government defends the target by accumulating or selling stocks 31% of the time, but because the target is so high public stock often reaches its capacity constraint and government is forced to let the price fall below the target price. Thus, this policy often fails to defend the price-band. However, this should not be seen as a failure. The price-peg programme is designed to maximise welfare. Hence failure to defend the price-peg is purposeful. The target price is a pretext for accumulating stocks to stabilise prices, and is not expected to be defended seriously. This behaviour shows that a price-band achieves reasonable welfare performance only by being designed in a way opposite to most expectations.

The optimal policy under commitment achieves the lowest coefficient of variation of price while requiring a lower average stock level than the discretionary policy (see Table 4). This is achieved by manipulating producers' expectations, which is confirmed by the fact that planned production is more variable under commitment than under discretion or any other policy. Stabilisation is more efficient under commitment because the policy maker, by manipulating expectations, can induce producers to stabilise in addition to public storage.

Table 4. Descriptive statistics on the asymptotic distribution

	Benchmark	Commitment	Discretion	Subsidy	Price-band
Coefficient of variation of price	0.206	0.168	0.169	0.174	0.177
Correlation between price and production	−0.810	−0.701	−0.717	−0.729	−0.765
Mean total stock	0.044	0.071	0.072	0.067	0.064
Coefficient of variation of planned production	0.023	0.028	0.026	0.026	0.026

Note: statistics calculated over 1,000,000 sample observations from the asymptotic distribution.

4.5 The sensitivity of time-consistency to supply elasticity

Since the difference between commitment and discretion is caused by manipulation of producers' expectations, we explore the sensitivity of this difference to supply elasticity, the most important parameter of producers' behaviour. Supply elasticity affects both the importance of time-consistency and the volatility without public policy (Table 5). A higher supply elasticity implies more stable prices and lower gains under a stabilisation policy since planned production reacts more to changes in expected prices. For a very low elasticity of 0.01, differences between commitment and discretion are negligible, since government cannot induce supply changes by manipulating expectations. The gains from commitment increase with the elasticity, but remain limited.

Table 5. Sensitivity to supply elasticity

Supply elasticity ($1/\mu$)	0.01	0.05	0.10	**0.50**	1.00	2.00
Total gains						
Commitment	0.084	0.084	0.084	**0.089**	0.097	0.104
Discretion	0.084	0.083	0.083	**0.084**	0.091	0.097
Coefficient of variation of price						
Without policy	0.230	0.227	0.223	**0.206**	0.197	0.188
Commitment	0.200	0.195	0.191	**0.168**	0.155	0.142
Discretion	0.200	0.196	0.191	**0.169**	0.156	0.143

Notes: Benchmark in bold. Welfare gains as a percentage of the steady-state commodity budget share.

5 Conclusion and perspectives

In this article, we evaluated the properties of different storage policies designed to maximise social welfare, in a model representing the cereals market in a self-sufficient developing country. Equilibrium without public intervention is non-optimal because markets are incomplete. We consider two state-contingent policies, with and without commitment, and two optimal simple rules of storage, a constant subsidy to private storage and a price-band defended by public stocks limited by a capacity constraint.

Among state-contingent storage rules, the policy under commitment achieves higher welfare results with lower average public stocks than the discretionary policy, because commitment induces producers to promote stabilisation. The gains numerically are small, however, since supply reaction is limited. Regarding simple rules of storage, the storage subsidy produces welfare results close to the discretionary policy, and the welfare gains under the price-band scheme are the lowest of all tested policies, but still represent 71% of the gains from the Ramsey-optimal policy.

A price-band policy is defined by three parameters: a floor price, a ceiling price and a capacity constraint on public storage. The parameters maximising welfare are a floor price equal to the ceiling price and close to the non-stochastic steady state, and a capacity constraint of 11% of steady-state production. This price-peg cannot be defended indefinitely since, at some point in time, the public stock will be exhausted or will reach its capacity constraint. The target price is set so high that the capacity constraint is most often binding and government is forced to let prices decrease below the intervention level. The target price is not meant to be seriously defended, instead it is a way to force the accumulation of public stocks above the private stock level without intervention.

The small welfare differences found between policies under commitment and discretion contrast with the accounts of food price policies in developing countries (Poulton et al., 2006, Tschirley and Jayne, 2010), where discretionary policies have proven very costly. Our idealised setting differs from real situations in at least one crucial aspect. In reality, the policy maker is not a benevolent planner. There are political cycles and people expect different governments to follow different policies (Alesina and Gatti, 1995). There are rent-seeking behaviours, which may create uncertainty about the group obtaining eventually the most important weight in the political process. These considerations, traditionally absent in the design of optimal dynamic policies, weaken the case for discretionary policies and reinforce the need to study simple rules of stabilisation whose management can be decentralised to an independent agency.

The potential role of simple storage rules in stabilisation policies may be based also on considerations of model uncertainty. Volatility in agricultural prices is certainly caused by shocks other than just productive shocks. Demand shocks, monetary shocks and input price shocks are all candidates. Although a model featuring productivity shocks associated with private storage looks to be a good approximation for commodity price dynamics (Cafiero et al., 2011), there are still uncertainties about the true data generated process. There are so many uncertainties related to what are the appropriate model and the value of behavioural parameters that it might be better to rely on simple rules with good performance in a variety of settings, than on a fully optimal policy designed for a specific model, but which may behave poorly in other contexts.[9]

Appendices

A First-order conditions for the optimal policy under commitment

To solve the optimal policy problems presented above, we need to reformulate the complementarity equation (6) since it cannot enter directly as a constraint in a maximisation problem. To restate this equation, we introduce a positive slack variable, ϕ, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta \, \mathrm{E}_t \left(P_{t+1} \right), \tag{25}$$

$$S_t^{\mathrm{s}} \phi_t = 0. \tag{26}$$

[9]This is one usual justification for focusing on simple rules in the conduct of monetary policies (Taylor and Williams, 2010).

ζ is taken to equal zero since there is no subsidy under a public storage policy.

We follow Marcet and Marimon (1999) and express the optimal policy problem under commitment as a saddle-point functional equation problem

$$\begin{aligned} J\left(A_t^{\mathrm{T}}, \lambda_{t-1}+\nu_{t-1}\epsilon_t\right) = \min_{\Phi_t} \max_{\Omega_t} \Big\{ & v\left(P_t, Y\right) + w\left[P_t A_t^{\mathrm{T}} - \Psi\left(H_t\right) - \left(P_t+k\right)\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}}\right)\right] \\ & + \lambda_t\left(\phi_t - P_t - k\right) \\ & + \delta_t S_t^{\mathrm{S}} \phi_t \\ & - \nu_t \Psi'\left(H_t\right) \\ & + \left(\lambda_{t-1}+\nu_{t-1}\epsilon_t\right) P_t \\ & + \chi_t\left[A_t^{\mathrm{T}} - D\left(P_t\right) - S_t^{\mathrm{S}} - S_t^{\mathrm{G}}\right] \\ & + \beta \, \mathrm{E}_t\left[J\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}} + H_t\epsilon_{t+1}, \lambda_t + \nu_t\epsilon_{t+1}\right)\right] \Big\}, \end{aligned} \tag{27}$$

where $\Phi_t = \{\lambda_t, \delta_t, \nu_t, \chi_t\}$ and $\Omega_t = \{S_t^{\mathrm{S}} \geq 0, H_t, P_t, S_t^{\mathrm{G}} \geq 0, \phi_t \geq 0\}$. Lagrange multipliers corresponding to forward-looking terms (λ and ν) are included as state variables to make the problem recursive. They represent the constraint for the policy maker to respect previous promises. Since both apply to price expectations, they can be summed together. Thus the system has two state variables, A_t^{T} and $\lambda_{t-1}+\nu_{t-1}\epsilon_t$.

From the first-order conditions and the envelope theorem, and after some manipulations we get the following system of complementarity conditions that, in addition to the transition equation (11), defines the dynamics of an optimal policy under commitment

$$S_t^{\mathrm{S}} : S_t^{\mathrm{S}} \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta \, \mathrm{E}_t\left(wP_{t+1} + \chi_{t+1}\right) + \delta_t\phi_t \leq 0, \tag{28}$$

$$H_t : \beta \, \mathrm{E}_t\left(\epsilon_{t+1}\chi_{t+1}\right) = \nu_t \Psi''\left(H_t\right), \tag{29}$$

$$P_t : v_P\left(P_t, Y\right) + wD\left(P_t\right) - \lambda_t + \lambda_{t-1} + \nu_{t-1}\epsilon_t - \chi_t D'\left(P_t\right) = 0, \tag{30}$$

$$S_t^{\mathrm{G}} : S_t^{\mathrm{G}} \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta \, \mathrm{E}_t\left(wP_{t+1} + \chi_{t+1}\right) \leq 0, \tag{31}$$

$$\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \delta_t S_t^{\mathrm{S}} \leq 0, \tag{32}$$

$$\lambda_t : \phi_t = P_t + k - \beta \, \mathrm{E}_t\left(P_{t+1}\right), \tag{33}$$

$$\delta_t : S_t^{\mathrm{S}}\phi_t = 0, \tag{34}$$

$$\nu_t : \beta \, \mathrm{E}_t\left(P_{t+1}\epsilon_{t+1}\right) = \Psi'\left(H_t\right), \tag{35}$$

$$\chi_t : A_t^{\mathrm{T}} = D\left(P_t\right) + S_t^{\mathrm{S}} + S_t^{\mathrm{G}}. \tag{36}$$

The time-inconsistency of the policy under commitment shows up in equation (30) in the lagged Lagrange multipliers. If the policy maker were to re-optimise after date 0, this would reset the Lagrange multipliers to zero.

B Characterisation of the optimal discretionary policy

For the discretionary problem, since we focus on a Markovian equilibrium, price can be characterised by a function of the state variable: $P_t = \psi\left(A_t^{\mathrm{T}}\right)$. Using this function ψ, the discretionary problem is formulated as a recursive problem similar to (27) except for the use of ψ to characterise price expectations and the absence of lagged Lagrange multipliers since there are no past promises to respect:

$$
\begin{aligned}
J\left(A_t^{\mathrm{T}}\right) = \min_{\Phi_t} \max_{\Omega_t} \Big(& v\left(P_t, Y\right) + w\left[P_t A_t^{\mathrm{T}} - \Psi\left(H_t\right) - \left(P_t + k\right)\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}}\right)\right] \\
& + \lambda_t \left\{\beta\, \mathrm{E}_t\left[\psi\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}} + H_t \epsilon_{t+1}\right)\right] + \phi_t - P_t - k\right\} \\
& + \delta_t S_t^{\mathrm{S}} \phi_t \\
& + \nu_t \left\{\beta\, \mathrm{E}_t\left[\psi\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}} + H_t \epsilon_{t+1}\right) \epsilon_{t+1}\right] - \Psi'\left(H_t\right)\right\} \\
& + \chi_t \left[A_t^{\mathrm{T}} - D\left(P_t\right) - S_t^{\mathrm{S}} - S_t^{\mathrm{G}}\right] \\
& + \beta\, \mathrm{E}_t\left[J\left(S_t^{\mathrm{S}} + S_t^{\mathrm{G}} + H_t \epsilon_{t+1}\right)\right] \Big).
\end{aligned} \tag{37}
$$

It is not possible to characterise the first-order conditions of this problem since we cannot assume ψ to be differentiable because of the occasionally binding constraints.[10]

C Equations of a price-band programme

For a rigorous mathematical characterisation of the behaviour of public stock under a price-band we need to introduce two variables: $\Delta S^{\mathrm{G}+}$ and $\Delta S^{\mathrm{G}-}$, which refer to increase and decrease in public stock. Both are positive and bounded from above. The increase in public stock is bounded from above by the remaining storage capacity, and the decrease in public stock by the level of existing stocks. To defend the price-band, public stocks are managed by the four conditions (20)–(23), which can be restated as two mixed complementarity equations:[11]

$$0 \le \Delta S_t^{\mathrm{G}+} \le \bar{S}^{\mathrm{G}} - S_{t-1}^{\mathrm{G}} \quad \perp \quad P_t - P^{\mathrm{F}}, \tag{38}$$

$$0 \le \Delta S_t^{\mathrm{G}-} \le S_{t-1}^{\mathrm{G}} \quad \perp \quad P^{\mathrm{C}} - P_t. \tag{39}$$

Equation (38) means that public stocks increase to prevent the price from decreasing below floor price, P^{F}. The floor is defended until public stocks reach the limit $\bar{S}^{\mathrm{G}}$. Equation (39) governs the decrease of public stocks. They decrease to prevent price from rising above the ceiling price, P^{C}. The release of stocks is constrained by the existing level of stocks S_{t-1}^{G}.

[10] In practice, since ψ is approximated by a spline, which is differentiable everywhere, we numerically solve the dynamic programming problem by solving the corresponding first-order conditions.

[11] Here the "perp" notation ($\perp$) is extended to situations with two complementarity constraints. The expression $a \le X \le b \ \perp \ F(X)$ is a compact formulation for $X = a \Rightarrow F(X) \ge 0, X \in]a, b[\Rightarrow F(X) = 0, X = b \Rightarrow F(X) \le 0$.

Market equilibrium and public stock transition are defined by

$$A_t^{\text{P}} = D(P_t) + S_t^{\text{S}} + \Delta S_t^{\text{G}+} - \Delta S_t^{\text{G}-}, \tag{40}$$

$$S_t^{\text{G}} = S_{t-1}^{\text{G}} + \Delta S_t^{\text{G}+} - \Delta S_t^{\text{G}-}. \tag{41}$$

The recursive equilibrium under a price-band programme is defined by the equilibrium equations (6), (8) and (38)–(40), and the transition equations (10) and (41).

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